

THREE ESSAYS ON DYNAMIC STOCHASTIC
GENERAL EQUILIBRIUM MODELS WITH
HETEROGENEOUS AGENTS AND FINANCIAL
FRICTIONS

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THREE ESSAYS ON DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM
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FRICTIONS

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The thesis consists of three essays.

The first essay develops a two-country heterogeneous-agents model with equilibrium default to explore the impact of financial integration between emerging countries and the U.S. The model shows that inefficient credit monitoring in emerging countries makes the borrowers in these countries more prone to default. The higher default risk makes financial intermediation in emerging countries less efficient (e.g. higher interest rate spread, higher default rate and lower borrowing capacity). Thus, households in emerging countries rely more on their own savings to hedge against future uncertainty. As a result, these countries have higher saving rate and lower saving return than the U.S. Given this logic, once funds are allowed to move across borders, money will move from emerging countries to the U.S seeking higher return. Thus, in the long run, the U.S gradually accumulates foreign liability along with depressed interest rate and relaxed credit limit. Meanwhile, the wealth inequality of the U.S gradually increases, whereas the consumption inequality in the U.S is mitigated due to the expanded consumer credit. The results are opposite for emerging countries.

The second essay uses the modeling framework developed in the essay One to draw important policy lessons pertaining to how an emerging country should

liberalize its capital account from an initial state of financial autarky. The model shows that, due to the inefficient financial intermediation, financial opening up by emerging countries may trigger a capital outflow in the short run. The sudden capital outflow raises the interest rate and crowds out domestic credit in emerging countries, and therefore a fraction of households in these countries become financially distressed, potentially leading to a liquidity crisis. The paper then shows that financial integration has different welfare impacts across households. For example, in emerging economies, rich households benefit from the financial integration but poor suffer. Gradual change in financial openness mitigates these differences leading to a higher overall welfare. Accordingly, the paper argues for a more gradual approach to capital account opening for emerging countries.

The third essay explores the linkage between financial disruptions and business cycles by studying the full equilibrium dynamics of an economy with two regimes, "normal business cycles" and "financial disruptions". The system behaves differently across the two regimes. During normal cycles, the economy is fluctuating around the center of the stochastic steady state where agents are able to maintain optimal capital stock through collateral borrowing. During the episodes of financial disruptions, the productive agents are financially constrained and the economy may deviate from its efficient state, followed by a sharp decline in output and capital price as well as a joint increase in risk premium and the Sharpe ratio. The basic mechanism of the model is the following: since the return on capital is higher if it is owned by high-productivity agents, in equilibrium high-productivity agents accumulate capital stocks through leverage. Due to the debt enforcement problem, there is a maximum level of leverage determined by the financial market, which depends on the market's projection

of the future value of collateral. The equilibrium leverage of high-productivity agents occasionally hits the endogenous maximum level, in which case financial disruptions occur. Because of the precautionary motive, there is only a low probability that the leverage constraint binds, while the absence of constraint characterizes the economy most of the time. Therefore, the likelihood of financial disruption depends on the history of macro shocks and individual actions that affect the equilibrium leverage. In other words, financial disruptions are endogenous rare episodes evolved over business cycles.

BIOGRAPHICAL SKETCH

After obtaining a bachelor degree in the University of Hong Kong, Tianli Zhao came to Cornell University to continue his education right away. During his first two years as a graduate student, Tianli received rigorous theoretical training and passed the PhD qualifying exams for both the Department of Economics and the Dyson School of Applied Economics and Management. In the meantime, Tianli cultivated his strong interest in using numerical algorithms to solve complex macroeconomics problems and discovered his talent in programming. Thus, he decided to pursue his PhD degree in the field of economics where he concentrated on international finance, macroeconomics and computational economics. As a PhD student, Tianli served as research assistant for several faculty members such as Professor Assaf Razin, Professor Gregory Poe and Professor Richard Boisvert. He also served as teaching assistant for a wide range of courses, from microeconomics theory to international finance, from introductory-level undergraduate courses to PhD core courses.

To my family.

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My appreciation extends to my classmates and friends whom I met in Ithaca. Although I could not mention everyone, there are many people who supported me throughout the years. I would like to express my sincerest appreciation to all these people.

Finally, I am especially indebted to my family in China for their kindness, unconditional love and support. Another Cornell graduate, Yan Qu, who then became my wife, continuously supported me all ways and all the time. Without her accompany, I would never have managed this venture.

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CHAPTER 1

GLOBAL IMBALANCE – THE LONG RUN IMPACT OF GLOBAL FINANCIAL INTEGRATION

1.1 Introduction

Since the sharp increase in global financial integration in the early 1990s, the United States has been gradually accumulating net foreign liabilities, which now equal nearly 30% of the U.S. GDP. Meanwhile, the holding of the U.S. assets by emerging countries has increased, resulting in global imbalances.

What causes such large capital flows from emerging countries to the United States? This question is of great importance. Not only because the resulting low interest rates and excessive borrowing by households in the United States helped to cause the 2008 financial crisis (Perri and Quadrini 2013), but also the ongoing global imbalance is expected to remain in the foreseeable future. Based on International Monetary Fund (IMF) estimates, the U.S. net foreign liability is expected to reach 8% of the world's output in 2014 (IMF World Economic Outlook, October 2012).

In the literature, several factors are cited to explain global imbalances, including differences in productivity growth (Hoffmann, Krause and Laubach, 2011), precautionary reserves (Carroll and Jeanne, 2009), demographic dynamics (Backus, Cooley and Henriksen, 2013), and valuation effects (Obstfeld and Rogoff, 2005). In addition, some studies explain global imbalances by reference to financial markets. For example, Caballero, Farhi, and Gourinchas (2008) rationalize global imbalances as an equilibrium outcome when separate regions

differ in their capacity to generate financial assets from real investments. Mendoza, Quadrini, and Rios-Rull (2009) attribute global imbalances to differential enforcement of financial contracts.¹ Panousi and Angeletos (2011) attribute global imbalances to varying levels of uninsurable idiosyncratic risks across countries.

This paper explains global imbalances from a new aspect of financial markets, namely cross-country differences in loan default rate caused by different credit-monitoring efficiencies. Credit monitoring provides information about an individual's credit history, including volume of borrowing and payments, late payments, and default. This information is important because the credit market can use it to identify defaulted households and withhold credit from those would-be borrowers. In some countries, however, such credit information is not fully available. As shown in Figure 1.1, there is a wide cross-country gap in the availability of credit information. This gap is especially prominent between relatively underdeveloped credit markets in emerging countries and the United States.

This paper uses a two-country heterogeneous-agents model with equilibrium default to show that differences in credit-monitoring efficiency between emerging countries and the United States could account for the global imbalance. The basic mechanism of the model involves three steps:

First, I show that, at the individual level, credit-monitoring efficiency affects default decisions of the individual household. To do so, I explicitly model equilibrium defaults as optimal choices made by some borrowers. Specifically,

¹Specifically, they assume that borrowers in separate countries can divert different fractions of individual income from creditors.

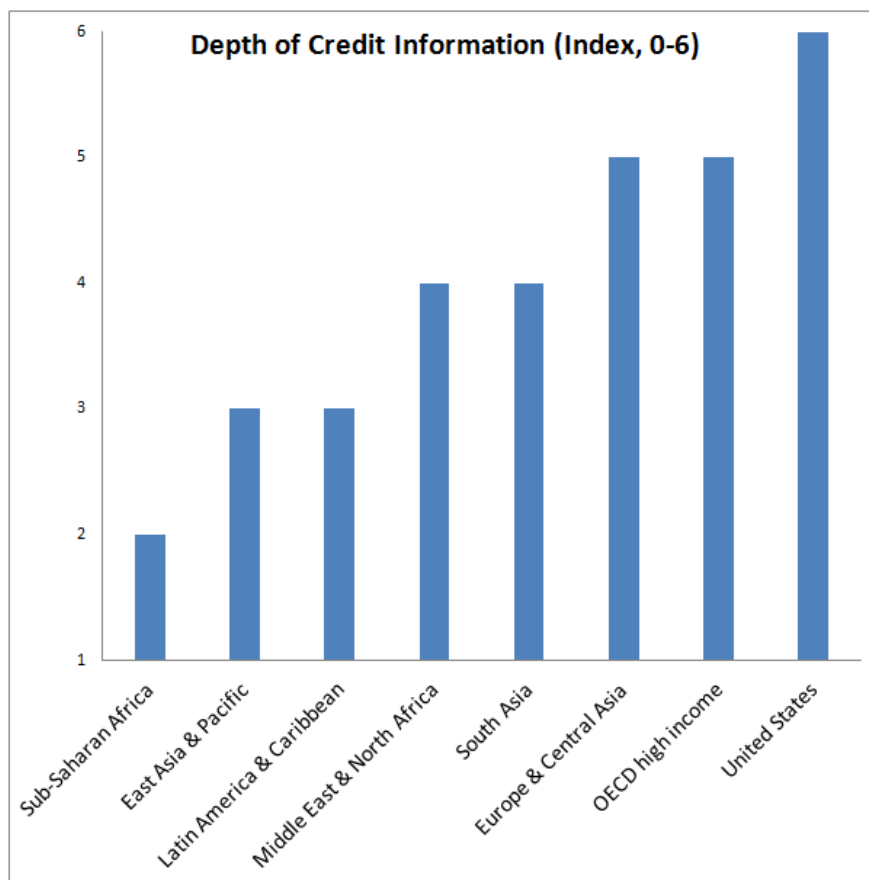


Figure 1.1: Depth of Credit Information

Source: World Bank Doing Business dataset. The depth of credit information index measures the coverage, scope, and accessibility of credit information available through either a public credit registry or a private credit bureau.

an indebted household faces a trade-off associated with default: if it defaults, it might be monitored due to a poor credit record and subsequently excluded from future access to the credit market.² On the other hand, default gives the indebted household the benefit of being released from its debt obligation via its bankruptcy filing. In a country with less efficient credit-monitoring, a defaulted household is less likely to be discriminated against in the future, leading to a

²Such a default penalty is consistent with bankruptcy filing under Chapter 7 of the U.S. Bankruptcy Code.

higher expected value of default. Such a household is more likely to default for any given debt contract.

Second, at the country level, credit-monitoring efficiency affects credit market equilibrium. Specifically, I introduce a competitive financial intermediary that offers a comprehensive schedule of lending rates, taking into account the likelihood of default.³ The financial intermediary responds to the higher default likelihood with a higher interest rate spread and tighter borrowing capacity. Consequently, *ceteris paribus*, in a country with less efficient credit monitoring, borrowing is more difficult and households must rely more on their savings to hedge against future uncertainty, leading to a higher saving rate and a lower return on savings in equilibrium.

Third, the varying levels of credit-monitoring efficiency between emerging countries and the United States could account for international capital flow during financial integration. In the long run, the United States (or emerging countries) gradually accumulates foreign liabilities (or assets) along with declining (or rising) interest rates and an expanded (or contracted) credit limit.

This paper contributes to the literature by establishing a deep micro-foundation for explaining the linkage between credit monitoring and household default decisions while allowing for equilibrium default.⁴ This not only makes the model more realistic, it also enables me to explain cross-country differences

³Such a contractual debt arrangement is similar to the arrangement modeled in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Arellano (2008).

⁴Prior studies do not allow borrowers to default, or they assume that the financial market can fully eliminate default so that default never occurs in equilibrium. However, there is compelling evidence that default always occurs. Moreover, the default rate and the interest rate spread are persistently higher in emerging countries than in the U.S.

in factors such as the default rate, the interest rate spread, and credit capacity, that have not been explained in prior studies (see Appendix).⁵ All of these factors are important for predicting the impact of financial integration between emerging countries and the United States.

Moreover, the use of an incomplete-market heterogeneous-agent framework enables the model to characterize the evolution of wealth distribution during financial integration. The model shows that the U.S. wealth inequality increases after financial integration, whereas the U.S. consumption inequality is mitigated due to the expansion of consumer credit for households in the United States, consistent with empirical evidence.⁶

The rest of the paper is organized as follows: Section 1.2 presents the theoretical model for a closed economy, Section 1.3 characterizes the autarky equilibrium and shows how monitoring efficiency affects the domestic credit market equilibrium, Section 1.4 extends the model to a two-country world and explores the impact of financial integration between emerging countries and the United States, and Section 1.5 concludes.

⁵For example, the model predicts that the credit market in the U.S has a lower interest rate spread, a lower default rate, and higher borrowing limits than emerging countries, consistent with stylized facts.

⁶For the empirical findings on the evolution of wealth inequality and consumption inequality, see Blundell, Pistaferri and Preston (2008), Diaz-Gimenez, Glover and Rios-Rull (2011), Hasset and Mathur (2012) and Wolff (2010). In another relevant paper, Armour, Burkhauser and Larrimore (2013) argue that incorporating accrued capital gains to measure yearly changes in wealth would dramatically reduce the observed growth in the U.S. inequality.

1.2 The Theoretical Model

This section constructs the model of optimal default and endogenous debt contracts in the context of a closed economy. There are three sectors: a production sector, a household sector and a financial intermediary sector.

Environment and Demographics:

In this study's theoretical model, the economy consists of a large number of heterogeneous households with the same preferences, but household labor productivity is subject to idiosyncratic shocks. As in Chatterjee, Corbase, Nakajima, and Rois-Rull (2007), during each specified time period, a fraction $(1 - \rho)$ of the population in each country will die and the same number of newborn will enter the economy with clean financial records.⁷ Thus, the total population for each country is constant.

Production Sector:

In the baseline model, I assume a competitive production sector that produces consumption goods using linear technology:

$$y_t = \int \theta_t(s_t) \bar{n} d\mu(s_t) \quad (1.1)$$

where $\theta_t(s_t)$ is idiosyncratic labor productivity and $\mu(s_t)$ is the population measure at the current level of productivity s_t . To keep the analytical model straightforward, I assume for now that the productivity shock is independent and identically distributed (*i.i.d*) with log-normal density. For the quantitative analysis, I allow these idiosyncratic household productivity shocks to follow a first order

⁷For simplicity, I assume the wealth of each dying agent, positive or negative, will be inherited by each newborn.

autoregressive (AR(1)) process. \bar{n} is the individual labor endowment, which is normalized to be one for all households. For the quantitative analysis, I allow the labor endowment to vary across households to match the household income distribution. In equilibrium, each unit of labor is paid at its marginal product, which is just equal to the realization of productivity:

$$w_t(s_t) = \theta_t(s_t) \quad (1.2)$$

Households:

Taking into account the possibility of death, the individual household preferences are given by the expected value of a discounted sum of monetary utility functions:

$$E_0\left\{\sum_{t=0}^{\infty}(\beta\rho)^t u(c_t(s_t))\right\} \quad (1.3)$$

where $0 < \beta < 1$ is the discount factor, and $c_t(s_t)$ is consumption in period t when state s_t is realized. All households have the same standard CRRA utility function $u(c_t(s_t)) = \frac{c_t(s_t)^{1-\gamma}}{1-\gamma}$, which is strictly increasing, concave, and differentiable. Households borrow and save by means of trading bonds. Similar to Chatterjee, Corbase, Nakajima and Rois-Rull (2007), I posit a market arrangement whereby unsecured loans of different sizes are treated as distinct financial assets. Therefore, the budget constraint faced by each household who chooses to honor its debt is given by equation (4):

$$c_t + q_t(b_{t+1})b_{t+1} = b_t + w_t(s_t), \quad (1.4)$$

where b_{t+1} is the bond position of the household (negative b_{t+1} corresponds to debt liability), and $q_t(b_{t+1})$ is the bond price, which is a function of loan size b_{t+1} .

Including a default option in the model necessitates departing from the conventional saving problem. At the beginning of each period, a household can choose whether to honor its debt. If the household chooses to default, then it is released from its liability (i.e., bond position “ b ” is set back to zero), but the household has the probability λ of being reported as having a bad credit history (given a default flag) by the countrys credit-monitoring system. Once a household is given a default flag, it will be penalized (or discriminated against) by the credit market in the future.

To make the model realistic, I incorporate both pecuniary and non-pecuniary default costs. Here I assume a specific type of non-pecuniary cost against a defaulted household; that is, such a household is prohibited by the credit market from borrowing. Such a non-pecuniary default cost is commonly seen in credit markets and is widely adopted by studies on default decisions (see, for example, Fay et al., 1998; Chatterjee et al., 2007; and Athreya et al., 2012). Here the pecuniary cost of default (such as higher borrowing premium and auto insurance premiums) is modeled as it is in Chatterjee, Corbase, Nakajima, and Rois-Rull (2007), who assume that a household with a default flag experiences a loss equal to a fraction ϕ of its earnings. Moreover, I assume that the defaulting agent will experience the worst income realization during the default period. This assumption captures actual market observations whereby an agent who files for bankruptcy protection can consume only his basic salary (amounting to the lower bound of income realization) for that period.

Note that a higher λ implies that a defaulted household is more likely to be discriminated against in the future. Therefore, the parameter λ characterizes the market’s credit-monitoring efficiency.

To formalize the households problem recursively, let $V(b, s, \lambda)$ denote the optimal expected lifetime utility of a household with a clean financial record, which has bond position b and current income state s . Note that the households value function $V(\cdot)$ also depends on credit-monitoring efficiency λ , as it affects the households optimal default choice.⁸ Specifically, the default decision is characterized by equation (5):

$$V(b, s, \lambda) = \max\{V_c(b, s, \lambda), \lambda[u(\underline{y}) + \beta\rho E\{V_d(0, s', \lambda)\}] + (1 - \lambda)V(0, s, \lambda)\}. \quad (1.5)$$

As shown in equation (5), an indebted household faces a trade-off associated with its default action. The first term on the right-hand-side $V_c(b, s, \lambda)$ denotes the optimal expected lifetime utility of the household if he chooses to honor his debt contract in the current period. In this case, he faces the following problem:

$$V_c(b, s, \lambda) = \max_{c, b'} \{u(c) + \beta\rho \sum_{s'} p(s') V(b', s', \lambda)\} \quad (1.6)$$

subject to the recursive borrowing constraint:

$$c(b, s, \lambda) + q(b', \lambda)b'(b, s, \lambda) = b + w(s). \quad (1.7)$$

The second term on the right-hand-side, $\lambda[u(\underline{y}) + \beta\rho E\{V_d(0, s', \lambda)\}] + (1 - \lambda)V(0, s, \lambda)$ gives the expected value of default. If the indebted household defaults, the default action gives it the benefit of having its debt obligation released via a bankruptcy filing. On the other hand, it is possible (i.e., with probability λ) that the defaulting household will be reported as having a poor credit record. In this case, the household can only consume its lowest income realization \underline{y}

⁸Note that λ is a time-invariant parameter under this setup. For notational convenience, it is specified as a function argument.

during the default period, and then it will be subsequently excluded from future access to the credit market. Formally, $u(\underline{y}) + \beta\rho E\{V_d(0, s', \lambda)\}$ is the value of a defaulting household reported with a bad credit record (default flag). Once given the default flag, the household is partially excluded from the credit market, i.e., it can save but it cannot borrow. The household then faces the following problem in the future:

$$V_d(b, s, \lambda) = \max_{c_d, b'_d} \{u(c_d) + \beta\rho \sum_{s'} p(s') V_d(b', s', \lambda)\} \quad (1.8)$$

subject to:

$$c_d(b, s, \lambda) + q(b', \lambda)b'_d(b, s, \lambda) = b + \phi w(s). \quad (1.9)$$

and an additional no-borrowing constraint: $b'_d \geq 0$. Because of the inefficient credit monitoring, it is also possible (i.e., with probability $1 - \lambda$) that the defaulted household will not be reported with a poor credit record. In this case, the household is able to get rid of its debt obligation while maintaining a clean credit record (so that its value becomes $V(0, s, \lambda)$).

The household chooses to default whenever its expected value of default is higher than the value of honoring its debt obligation, that is: $V_c(b', s', \lambda) < \lambda[u(\underline{y}) + \beta\rho E\{V_d(0, s'', \lambda)\}] + (1 - \lambda)V(0, s', \lambda)$. Whether a household will default depends on the realization of s' ; hence, it is unknown at the time when the bond contract b' is issued. To derive the probability of default, let $D(b', \lambda)$ be the set of future productivity realizations s' for which default is optimal at bond position b' . Specifically,

$$D(b', \lambda) = \{s' \in S : V_c(b', s', \lambda) < \lambda[u(\underline{y}) + \beta\rho E\{V_d(0, s'', \lambda)\}] + (1 - \lambda)V(0, s', \lambda)\} \quad (1.10)$$

Then, the default probability $Pr(b', \lambda)$ can be characterized by the measure of

default sets:

$$Pr(b', \lambda) = \mu(D(b', \lambda)). \quad (1.11)$$

A typical household's policy functions and value functions are illustrated in Figure 1.2.⁹ Note that the "jumps" on the consumption curves (panel 1) and saving curves (panel 2) occur when households choose to default. When a household decides to default, its debt position will be cleared to zero in the next period.¹⁰ Consequently, the defaulted household (if caught by the credit-monitoring system), will be excluded from future borrowing. Thus, its future consumption/saving decisions follow the new policy rule (panels 3 and 4). Moreover, the inclusion of the default option also leads to the non-concavity of the value functions (panel 5)

Risk-neutral competitive financial intermediaries:

In each country, there are competitive risk-neutral banks. Each bank can borrow or lend to other banks at a risk-free interbank interest rate. Each bank maximizes the present discounted value of current and future cash flows:

$$\max \sum_{t=0}^{\infty} R_f^{-1} \pi_t. \quad (1.12)$$

$$\pi_t = \int (1 - Pr(b_t, \lambda)) b_t d\mu(s_{t-1}, b_t) - \int q(b_{t+1}, \lambda) b_{t+1} d\mu(s_t, b_{t+1}), \quad (1.13)$$

where $(1 - Pr(b_t, \lambda))$ is the probability of receiving the repayment for the partic-

⁹For visibility, only a subset of all possible states are plotted.

¹⁰Note that the plotted consumption curves in the default case are not necessarily the actual policy. The actual consumption during the default period depend on whether they are reported as having a bad credit record by the monitoring system.

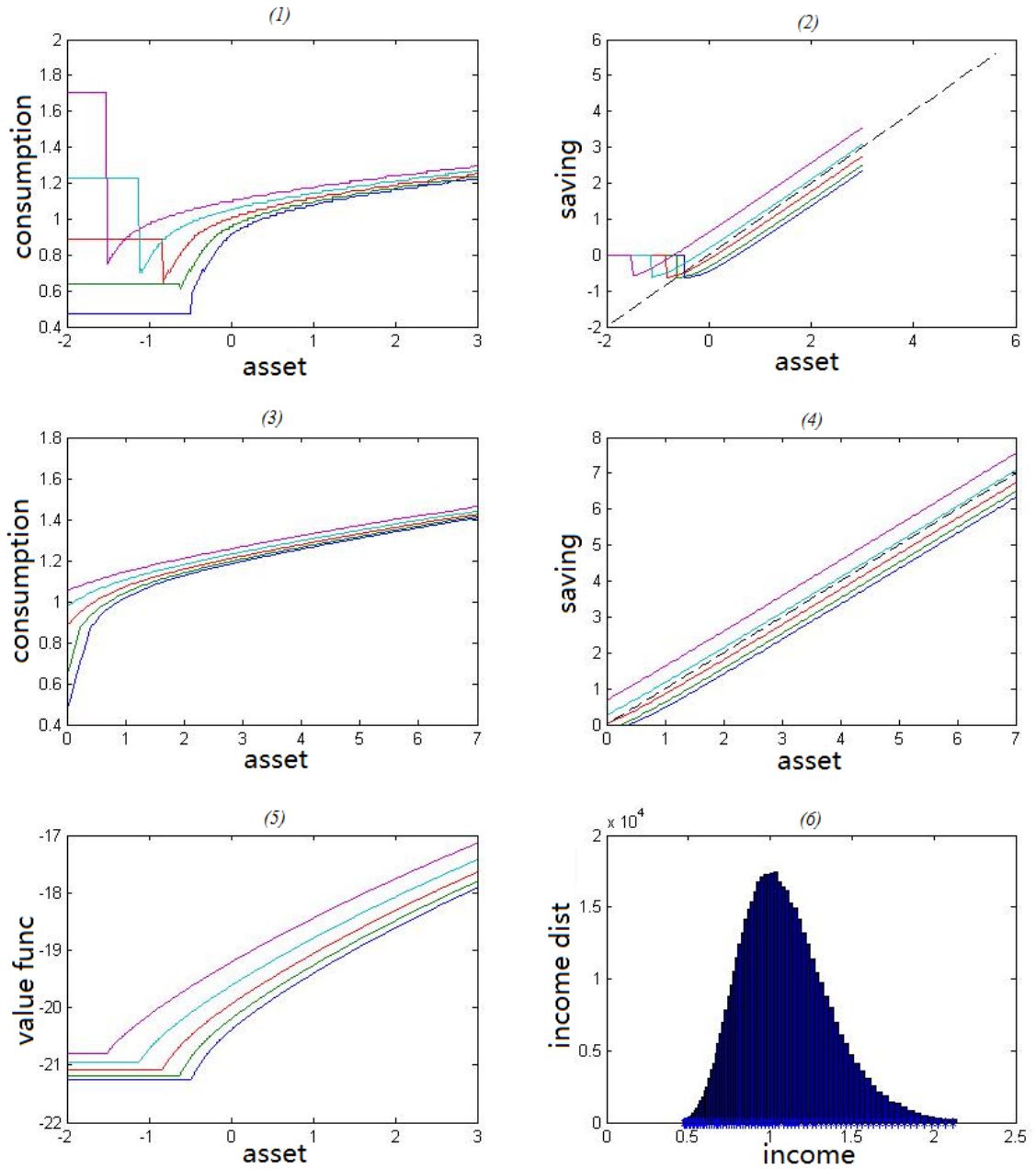


Figure 1.2: Policy Functions, Value Functions and Income Distribution

Panel (1)-Consumption Function; Panel (2)-Saving Function; Panel (3)-Consumption policy with default flag; Panel (4)-Saving policy with default flag; Panel (5)- Value function; Panel (6)- Income Distribution

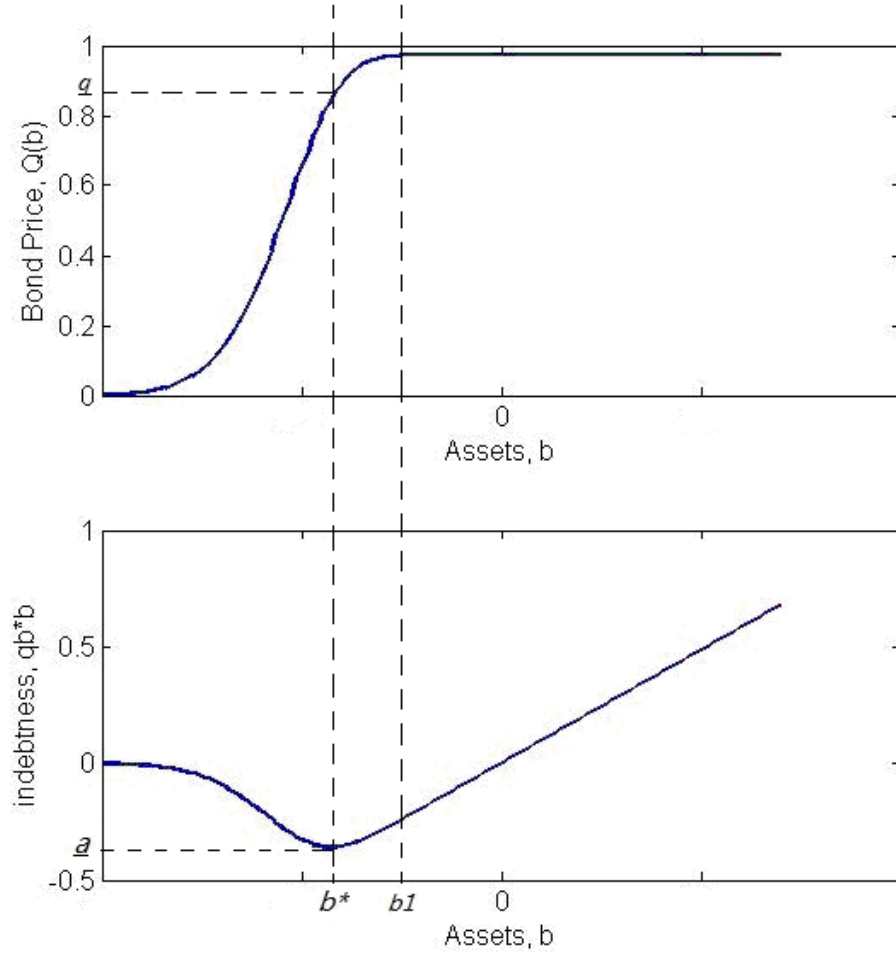


Figure 1.3: Endogenous Bond Schedule and Borrowing Limit

ular loan contract.¹¹ The zero-profit condition implies that, under equilibrium, risk-neutral creditors set the price schedule as:¹²

$$q(b', \lambda) = \frac{1}{R_f}(1 - Pr(b', \lambda)) \quad (1.14)$$

¹¹If some borrowers do not survive to repay their loans and some depositors do not survive to collect on their deposits, some newborns will inherit their debt/asset positions. So in such cases banks do not need to consider the survival probability.

¹²It is easy to see that any bond price other than $q(s, b', \lambda)$ will lead to either zero or an infinite supply of this particular bond contract.

The upper panel of Figure 1.3 illustrates an endogenous bond schedule as a function of the loan size (for a fixed value of λ). One can see that the bond price decreases with the total volume of debt. This is because households are more likely to default when they have more debt.¹³

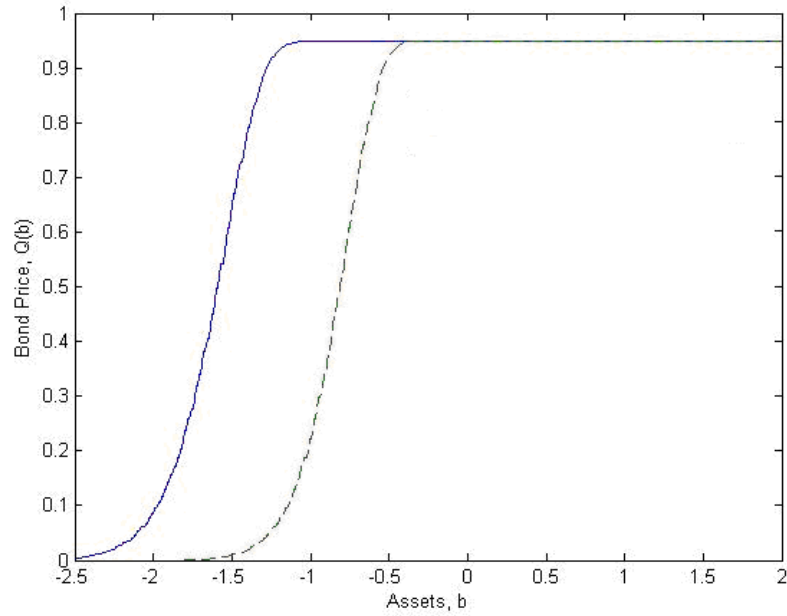
The lower panel of Figure 1.3 plots the total amount of indebtedness by household, which is the product of the bond price $q(b', \lambda)$ and the volume of bonds b' (i.e. $a = q(b', \lambda)b'$). As the bond price decreases with total volume issued, the household at some point is no longer able to accumulate debt by issuing more bonds. In other words, there is a limit to the household's indebtedness. In the example given in the lower panel of Figure 1.3, the household would never optimally choose a bond contract with $b < b^*$, because it can find an alternative contract that increases consumption today by the same amount while incurring lower liability for the next period. Therefore, the household's endogenous borrowing limit is given by $\underline{a} = q(b^*, \lambda)b^*$. Moreover, b_1 is the lowest asset position that is default-free. Thus, "risky loans" refer to bond contracts in the range of $b \in [b^*, b_1]$, which carry positive default premiums up to $1/\underline{q} - R_f$.¹⁴

The bond schedule $q(b', \lambda)$ is also a function of λ , which characterizes the efficiency of credit monitoring. A lower λ , which is associated with less efficient monitoring, implies that a defaulted household is less likely to be discriminated against in the future, leading to a higher expected value of default. Therefore, in a market with a lower λ , a household is more likely to default for any given debt level. Consequently, the financial intermediary responds to this higher default likelihood with a higher saving-loan spread as well as tighter borrowing

¹³The property is proved analytically in a similar setup by Eaton and Gersovitz, 1981; Chatterjee et al., 2007; and Arellano, 2007)

¹⁴ \underline{q} is the lowest bond price that can possibly be observed in equilibrium.

capacity (Figure 1.4).



Note that competitive financial intermediaries take the market risk-free rate as given. Thus, the results above constitute partial equilibrium outcomes. The following section characterizes the general equilibrium under which the risk-free rate is determined to clear the credit market for all heterogeneous households.

1.3 Characterizing Credit Market Equilibrium

This section characterizes the general equilibrium for a closed credit market in which the fund is not allowed to flow in and out of the country. It begins with the definition of such a stationary equilibrium. Then, the parameter values calibrated for the numerical solutions are discussed. With these parameter values, I used a reasonably calibrated example to demonstrate the impact of credit mon-

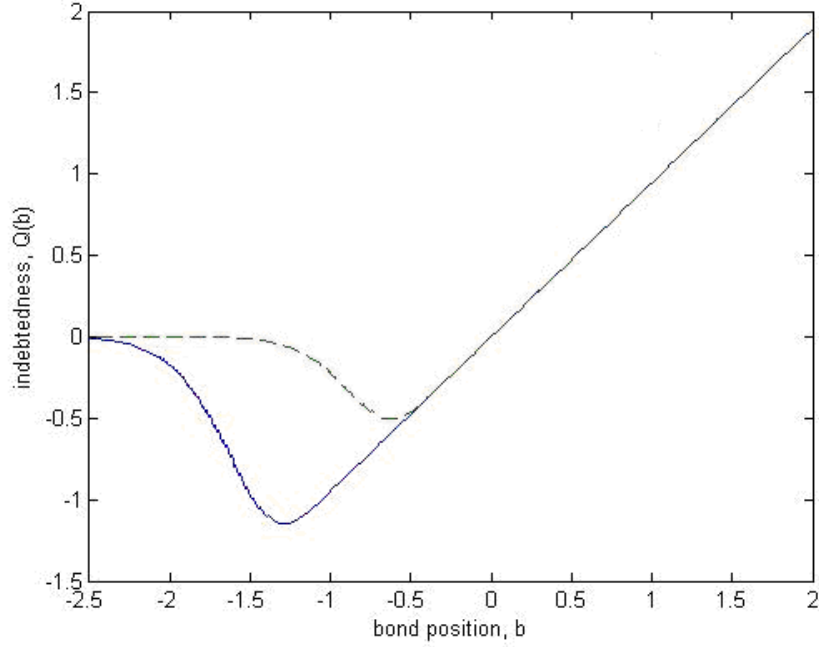


Figure 1.4: Bond Schedule and Borrowing Limit with Different λ .

Upper Panel: Bond Schedule. Lower Panel: Borrowing Limit. The solid line corresponds with λ_1 and the dashed line corresponds with λ_2 . $\lambda_1 > \lambda_2$

itoring on the domestic credit market.

The steady state equilibrium for a closed economy is defined as:

Definition 1 *Steady State Equilibrium for a Closed Economy:*¹⁵

With no international capital mobility, a steady-state competitive equilibrium for an economy with monitoring-efficiency λ is a set of strictly positive wage $w(s)$, the risk-free interest rate $R_f(\lambda)$, loan-price schedule $q(b', s, \lambda)$, strictly positive quantities of aggregate labor demand N , net interbank borrowing/lending B_{int} , and decision rules

¹⁵The equilibrium of a closed domestic credit market without international mobility of capital is similar to the economy studied by Chatterjee Corbase, Nakajima and Rois-Rull; the existence of such a steady state equilibrium is proven in Chatterjee et al 2007.

$c(b, s, \lambda, df)$, and $b'(b, s, \lambda, df)$,¹⁶ and a distribution of households $\mu(s, b, df)$ such that:

- (i) decision rules $c(b, s, \lambda, df)$ and $b'(b, s, \lambda, df)$ solve the household's optimization problem.
- (ii) aggregate labor demand N solves the firm's optimization problem.
- (iii) all bond contracts satisfy the intermediary's optimization problem.
- (iv) the labor market clears.
- (v) $B_{int} = 0$, interbank borrowing/lending clears domestically.
- (vi) the asset market clears: $\int q(b', s, \lambda) b'(b, s, \lambda, df) d\mu(s, b, df) = 0$.¹⁷

The computation procedures in this study are an extension of those put forth by Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) in their work on consumer credit with default risk. Computing the equilibrium requires four steps: an inner loop in which the household decision problem's given bond price schedules are solved; a middle loop in which the bond schedule converges given the risk-free interest rate; and an outer loop in which the market-clearing risk-free rate is obtained. Finally, the parameter values that yield equilibrium allocations with the desired properties are determined.¹⁸ The calibration of parameter values is discussed below.

¹⁶The fourth argument in the policy function is the default flag. $df=0$ means the households credit record is clean, while $df=1$ means a bad credit record. Thus, $c(b, s, \lambda, 1) = c_d(b, s, \lambda)$. Moreover, the default decision is embedded in the households consumption/saving policies.

¹⁷Note that under financial autarky, condition (v) and (vi) automatically imply each other.

¹⁸This paper's computation load is intensive. The numerical algorithm is coded in Matlab and Fortran-95

1.3.1 Calibration of Parameter Values:

The model is calibrated to match key the statistics for the U.S. As the benchmark economy, the U.S. economy is assumed to have the most efficient credit-monitoring system in the world, corresponding to the case in which $\lambda = 1$.

To enhance the quantitatively property of the model, the theoretical model discussed in Section 1.2 is generalized in the following two aspects.

First, to quantitatively match the income distribution of households in the United States, I allow the labor endowment η_j to vary across households.¹⁹ I divide households into seven groups each with different labor endowment. The population measure $\{\mu_{\eta_h}\}$ and the labor endowment $\{\eta_h\}$ for each group $h \in \{1, 2, \dots, 7\}$ are jointly calibrated to match the income distribution of the U.S. household.²⁰ (see Figure 1.6) Due to the homogeneous property of the CRRA utility, it is straightforward to show that the labor endowment η_j can be reduced from the endogenous state space by normalizing the bond position as the share of the expected household income.²¹ For notational convenience, I hereafter use the letter b_j denotes the normalized bond position as percentage of expected income for household j , while its labor endowment η_j is omitted.

Second, I added the persistence of the idiosyncratic shocks by assuming that

¹⁹One interpretation of this heterogeneity is that households are endowed with different talents. It is also very common that the aggregate labor units from all family members are varying across households.

²⁰The data on the income distribution of the U.S. household is from the US Census Bureau; Income, Poverty, and Health Insurance Coverage in the United States: 2011

²¹The policy space and the state space of the household with a higher labor endowment are just scaled replicas of the others.

Table 1.1: Parameters of Labor Endowment and Income Distribution

Statistics	Target	Model	Parameter	Value
% earning of the lowest 10%	1.07%	1.08%	η_1	0.1065
% earning of the lowest 20%	3.50%	3.54%	η_2	0.3084
% earning of the lowest 30%	7.28%	7.26%	η_3	0.6028
% earning of the lowest 40%	12.47%	12.39%	η_4	0.9673
% earning of the lowest 50%	19.15%	18.95%	η_5	1.4019
% earning of the lowest 60%	27.60%	27.43%	η_6	2.1730
% earning of the lowest 70%	38.31%	38.28%	η_7	3.1543
% earning of the lowest 80%	52.10%	51.93%	μ_{η_1}	0.1
% earning of the lowest 90%	70.25%	70.16%	μ_{η_2}	0.2
% earning of top 5%	18.52%	18.52%	μ_{η_3}	0.2
income GINI	0.4440	0.4442	μ_{η_4}	0.2
mean/median	1.33	1.33	μ_{η_5}	0.2
overall mean income	1.00	1.00	μ_{η_6}	0.15

the productivity of household j follows an log-AR(1) process:

$$\ln(\theta_t^j) = \mu(1 - \rho) + \rho \ln(\theta_{t-1}^j) + \sigma e_t^j,$$

where $e_t^j \sim i.i.d.N(0, 1)$. The expected individual log-productivity μ is normalized to one without loss of generality. The persistence parameter ρ and standard deviation σ are estimated by Heaton and Lucas (1996) on the basis of evidence from the Panel Study of Income Dynamics (PSID) for households in the United States.²² To numerically approximate the AR(1) process using Markov transitioning matrix, the continuous AR(1) process is discretized by a large number

²²Heaton and Lucas estimate the idiosyncratic income process per family member, therefore the estimated ρ and σ are consistent with the idiosyncratic productivity process in this

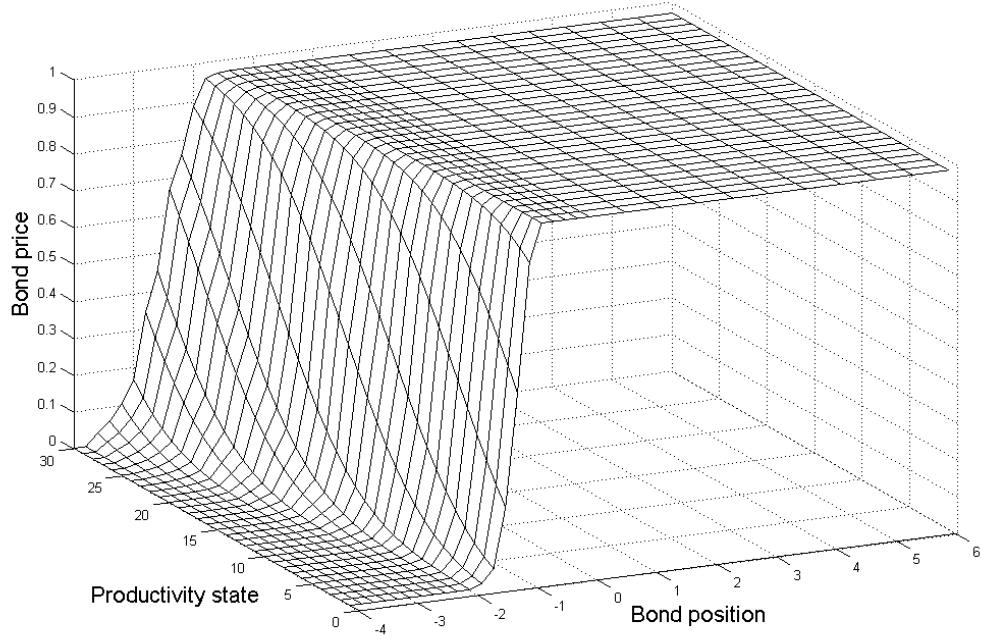


Figure 1.5: Bond Schedule for a Typical U.S. Household as a Function of the Household Productivity State s and Bond Volume b .

of productivity states using Gauss-Hermite quadrature.

It is worth noting that, due to the persistence of the productivity shocks, the default probability now becomes the conditional measure of the default set, which depends on the current productivity state. That is,

$$Pr(b', s, \lambda) = \mu(D(b', s, \lambda) | s).$$

By the same token, the bond schedule is also a function of current state: $q(b', s, \lambda) = \frac{1}{R_f}(1 - Pr(b', s, \lambda))$. Figure 1.5 plots the bond schedule for a U.S. household with $n = 1$.

Besides the parameters associated with idiosyncratic productivity process paper. The cross-sectional differences in households expected income are captured by the cross-households-variation of labor endowment η_j .

Table 1.2: Parameter Values

Notation	Parameter Name	Value	Targets and Sources
ρ	shock persistence	0.529	Heaton and Lucas (1996)
σ	shock standard deviation	0.251	Heaton and Lucas (1996)
β	time preference	0.969	Average Risk-Free rate = 3.30%
ϕ_1	pecuniary cost of default	0.039	the U.S. default rate = 1%
ρ	birth/death rate	0.02	Average adult life = 50 years
γ	relative risk-aversion	2	Standard value

and labor endowment distribution, there are other parameters to be determined. (Table 1.2)

The birth/death rate ρ is determined to match the average length of an adult life, taken to be 50 years. The pecuniary cost of having a bad credit record ϕ is set to target the default rate of U.S. households at 1%.

Moreover, I adopt standard time-separable constant relative risk-aversion preferences that are characterized by two parameters: the discount rate β and the risk-aversion coefficient γ . The coefficient of relative risk aversion γ is set to a standard value of 2. The discount rate β is calibrated to target the risk-free interest rate at the historical average risk-free interest rate in the United States before the era of financial integration.

All calibrations are performed jointly using the Simulated Method of Moments. Although the model is highly stylized, it matches some key features of the U.S. data.

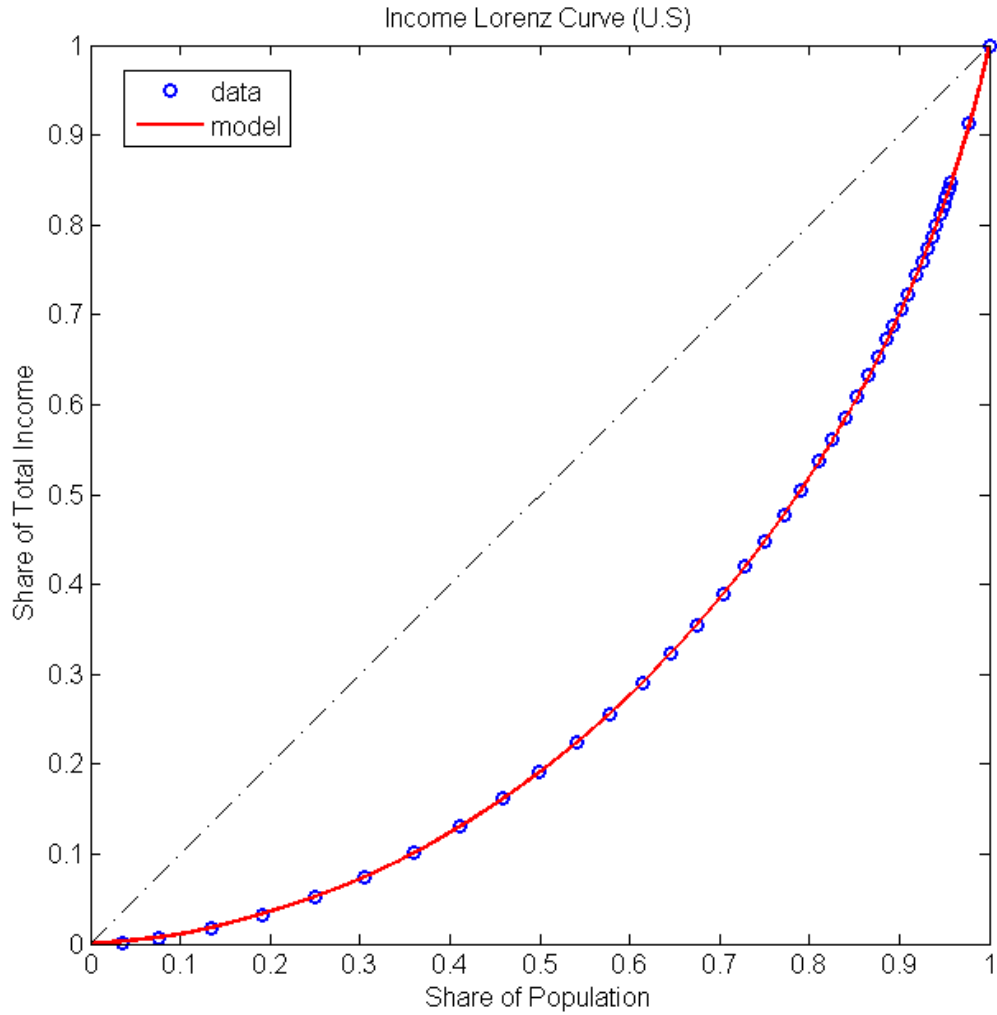


Figure 1.6: The Income Lorenz Curve (U.S). Model v.s Data

1.3.2 The Impact of Credit Monitoring on the Domestic Credit Market

As discussed at the end of the previous section, under partial equilibrium at a given risk-free interest rate, the financial intermediary responds to a greater default likelihood with a higher saving-loan spread and tighter borrowing capacity. Consequently, in a credit market with lower λ , borrowing is more difficult and households must rely on their own savings to hedge against idiosyncratic

income risk, leading to a higher saving rate.

Under general equilibrium, this higher saving rate, which is motivated by household precaution, results in a lower equilibrium return on savings. Moreover, I also show in Figure 1.7 and Figure 1.8 that other equilibrium variables such as the default rate, the loan-deposit spread, the borrowing limit, and mean and median loan sizes are all functions of λ . In short, the model clearly identifies a relationship between credit market equilibrium and credit monitoring efficiency.

As shown in the introduction, emerging countries typically are less efficient at monitoring credit compared with the United States. Thus, the model predicts higher loan-deposit spreads, higher default rates, and lower borrowing capacity in emerging countries than in the United States. All these results are consistent with the stylized empirical features mentioned in the introduction.

Because of these differences regarding domestic credit market equilibrium, money flows from emerging countries to the United States during the process of global financial integration. In the next section, the paper applies the above model to a two-country world with differential credit-monitoring efficiencies to study the impact of financial integration between emerging countries and the United States.

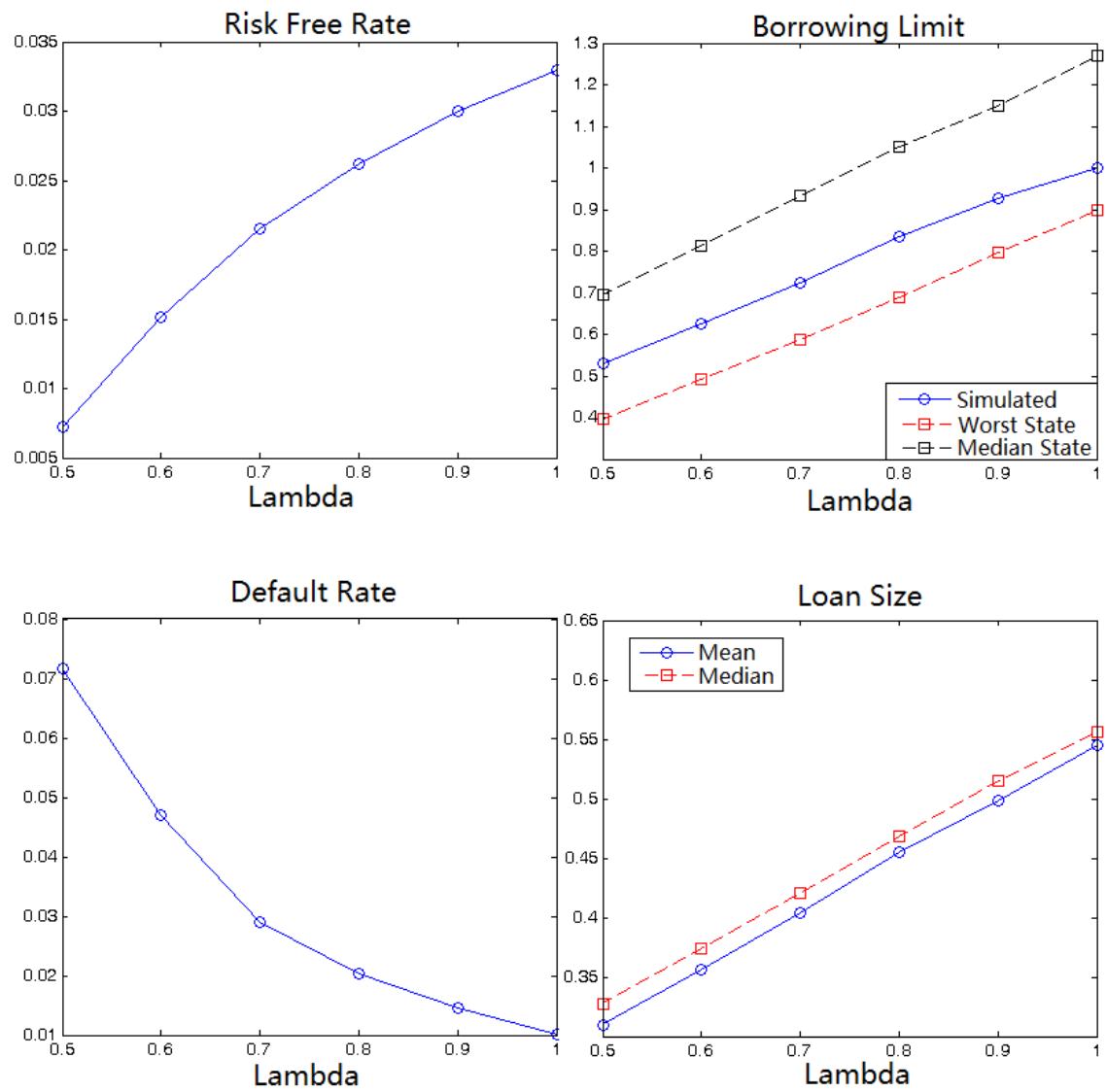


Figure 1.7: The Impact of Credit Monitoring on Domestic Credit Market Equilibrium

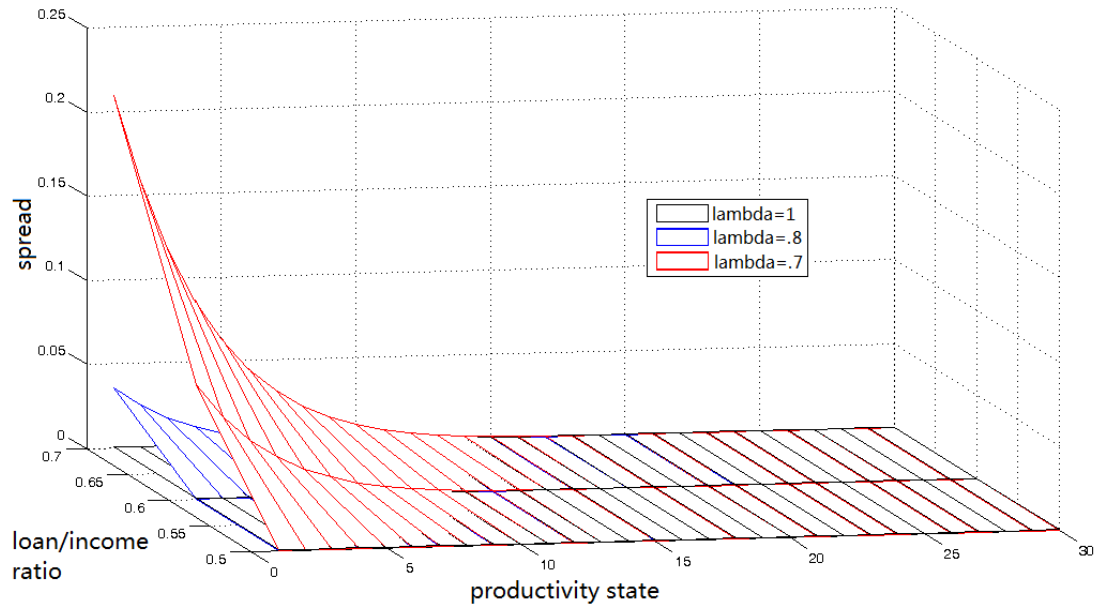


Figure 1.8: Loan-deposit-spread Given Various Credit Monitoring Efficiency λ

lambda=1 in black; lambda=0.8 in blue; lambda=0.7 in red.

1.4 Financial Integration Between Emerging Countries and the United States

After discussing domestic credit market equilibrium with no international capital mobility, Section 1.4 investigates the potential impact of financial integration between emerging countries and the United States. To do so, I applied the model to a two-country world in which countries C_1 and C_2 differ only in regards to their domestic credit-monitoring efficiency. C_1 (representing the U.S.) practices highly efficient credit monitoring, so households in C_1 , once in default, will forever carry poor credit records (i.e., $\lambda_1 = 1$). In comparison, C_2 (representing a set of key emerging countries) has a less developed credit-monitoring system. Therefore, defaulted households in C_2 are less likely to be monitored

with a default flag. (i.e., $\lambda_2 < \lambda_1$).²³ As explained in Section 1.3, without international capital mobility, the less efficient credit monitoring of emerging countries results a lower return on savings in these countries, compared with the savings in the United States.

Once two countries are financially integrated, however, money is able to move across their borders. Capital mobility therefore enables households to trade financial assets with both domestic and foreign financial intermediaries. Moreover, financial intermediaries can also borrow/lend with foreign counterparts via interbank loans. In other words, the credit market is now worldwide, instead of confined within each country. The definition of steady-state equilibrium with globally integrated capital markets is given below:

Definition 2 *Steady State Equilibrium with Integrated World Financial Markets:*

With perfect international mobility of capital, a steady-state competitive equilibrium is a set consisting of strictly positive wage $w^i(s)$, world-wide risk-free interest rate, R_f , loan-price schedules $q^i(b', s, \lambda_i)$, strictly positive quantities of aggregate labor demand N^i , net interbank borrowing/lending B_{int}^i , decision rules $c^i(b, s, \lambda_i, df)$ and $b^i(b, s, \lambda_i, df)$, and distributions of households $\mu^i(s, b, df)$ such that:

- (i) *decision rules $c^i(b, s, \lambda_i, df)$ and $b^i(b, s, \lambda_i, df)$ solve every household's problem in each country i .*

²³Following Mendoza et al. (2009), I assume the credit monitoring is residence-based. That is, residents in C_i are monitored by country i 's credit-monitoring system. Financial integration will not change a country's monitoring efficiency in the short run. This is a reasonable assumption because the enforcement and monitoring of loan contracts depends heavily on the local legal environment and market institutions.

- (ii) aggregate labor demand N^i solves the firm's optimization problem in each country i .
- (iii) all bond contracts satisfy the intermediary's optimization problem.
- (iv) the labor market clears in each country.
- (v) $\sum_{i=1,2} B_{int}^i = 0$, interbank borrowing/lending clears internationally.
- (vi) the asset market clears:²⁴ $\sum_{i=1,2} \int q^i(b', s, \lambda_i) b^i(b, s, \lambda_i, df) d\mu^i(s, b, df) = 0$.

With an integrated financial market, risk-free interest rates R_f are equated across countries.

1.4.1 Calibration of Credit-Monitoring Efficiency and Simulated Statistics

As shown in the introduction, there is a significant cross-country difference in credit-monitoring efficiency. The difference is especially prominent between the United States and emerging countries.²⁵ However, it is difficult to derive a direct mapping of these indicators to actual values of λ_i . Therefore, I follow Mendoza et al. 2009's pragmatic approach – I assume that the United States has the most efficient credit monitoring (i.e. $\lambda_1 = 1$) and then calibrate λ_2 for the emerging economies to match the U.S. Net Foreign Liability (NFL). Using the Simulated Method of Moments, the parameter λ_2 is calibrated to be 0.701 and the U.S.

²⁴Note that under financial autarky, condition (v) and (vi) automatically imply each other.

²⁵Besides the World Bank's Doing Business database, there are other indicators, such as the Financial Development Index (IMF 2006) and the index of financial liberalization (Abiad et al., 2008) all suggesting that the efficiency of financial intermediation differs significantly between the United States and emerging countries.

Table 1.3: Calibrate the Efficiency of Credit Monitoring

Notation	Parameter Name	Value	Targets and Sources
λ_1	prob of being monitored in C_1	1.00	The U.S. is assumed to have perfect credit-monitoring system
λ_2	prob of being monitored in C_2	0.701	Match the U.S. NFA position at S.S.

NFL is targeted at 22% of the U.S. GDP (Table 1.3).²⁶ The main statistics from numerical simulation of financial integration are shown in Table 1.4.²⁷

Before the integration of world financial markets:

The borrowing limit for households in the United States at the median productivity state is about 126% of their average labor income. In comparison, households in emerging countries have restricted access to credit markets. The borrowing limit for emerging country households at the median productivity state is about 94% of their average labor income. As a result, the risk-free interest rate is 115 basis points higher in the United States than it is in emerging countries. Moreover, the default rate of the U.S. households is targeted at 1%. The default rate in emerging markets is significantly higher, as households in these countries default on 3.07% of the loan contracts.

²⁶The imbalance between emerging countries and the United States is roughly 22% of the U.S. GDP.

²⁷Because the U.S. economy is approximately the same size as that of all key emerging markets combined, C_1 and C_2 are considered to be of the same size. All other parameters have the same value as those discussed in Section 1.3.

Table 1.4: Simulation Results (Steady State)

	Autarky		Integrated	
	C_1	C_2	C_1	C_2
Risk-Free Interest Rate	3.30	2.15	2.88	2.88
Borrowing Limit in \underline{s} (% of Expected Income)	90.03%	59.59%	101.08%	52.43%
Borrowing Limit in <i>median(s)</i> (% of Expected Income)	126.02%	94.26%	136.51%	84.95%
Borrowing Limit in \bar{s} (% of Expected Income)	181.03%	144.65%	188.21%	135.65%
max indebtedness (% of Expected Income)	100.01%	71.29%	110.61%	63.01%
median indebtedness (% of Expected Income)	57.12%	42.09%	65.12%	34.99%
mean indebtedness (% of Expected Income)	55.66%	40.09%	62.50%	34.27%
Wealth GINI	0.6464	0.5653	0.7032	0.5462
Consumption GINI	0.4304	0.4311	0.4307	0.4304
Default Rate (% of population)	1.00%	3.07%	1.02%	2.52%
Net Foreign Asset Position (% of the U.S. GDP)	N/A	N/A	-22.00%	22.00%

With integrated world financial markets:

In the steady state, the world risk-free interest rate is equalized at 2.88%. (Figure 1.10). Because the U.S. interest rate decreased during financial integration, U.S. households borrowed more from emerging countries, creating a negative Net Foreign Asset (NFA) position. Interestingly, the gap between credit accessibility in the United States and emerging countries has not narrowed but instead widened following global financial integration. For households in emerging countries, the borrowing limit is depressed by roughly 11%. Meanwhile, the borrowing limit for households in the U.S. rises by around 11%. As shown in Figure 1.11, the steady-state wealth distribution of the U.S. (emerging countries) in the integrated steady state shifts to the left (right) relative to those in the autarky steady state. In other words, following financial integration, households in the U.S. borrow more than before, while households in emerging countries save more than before. As a result, the United States gradually accumulates NFL, eventually reaching about 22% of the U.S. GDP.

It is also worth noting that financial integration affects the wealth distribution as well as consumption distributions. For the United States, the wealth inequality increases during the period of financial integration. The wealth GINI coefficient rises from 0.6464 to 0.7032. (See the left panel of Figure 1.9).²⁸ In comparison, the consumption inequality changes very little. (See the right panel of Figure 1.9). The consumption GINI coefficient rises slightly from 0.4304 to 0.4307. The larger increase in wealth inequality relative to consumption inequality is consistent with empirical observations (see for example, Blundell, Pistaferri and Preston, 2008; Budria, Diaz-Gimenez, Quadrini and Rios-Rull,

²⁸The wealth of household j at period t is the sum of its beginning financial position b_t^j and its income $w(s_t)\eta_j$. That is, $a_t^j(s_t) = b_t^j + w(s_t)\eta_j$.

2002; Hassett and Mathur, 2012; and Wolff, 2010).

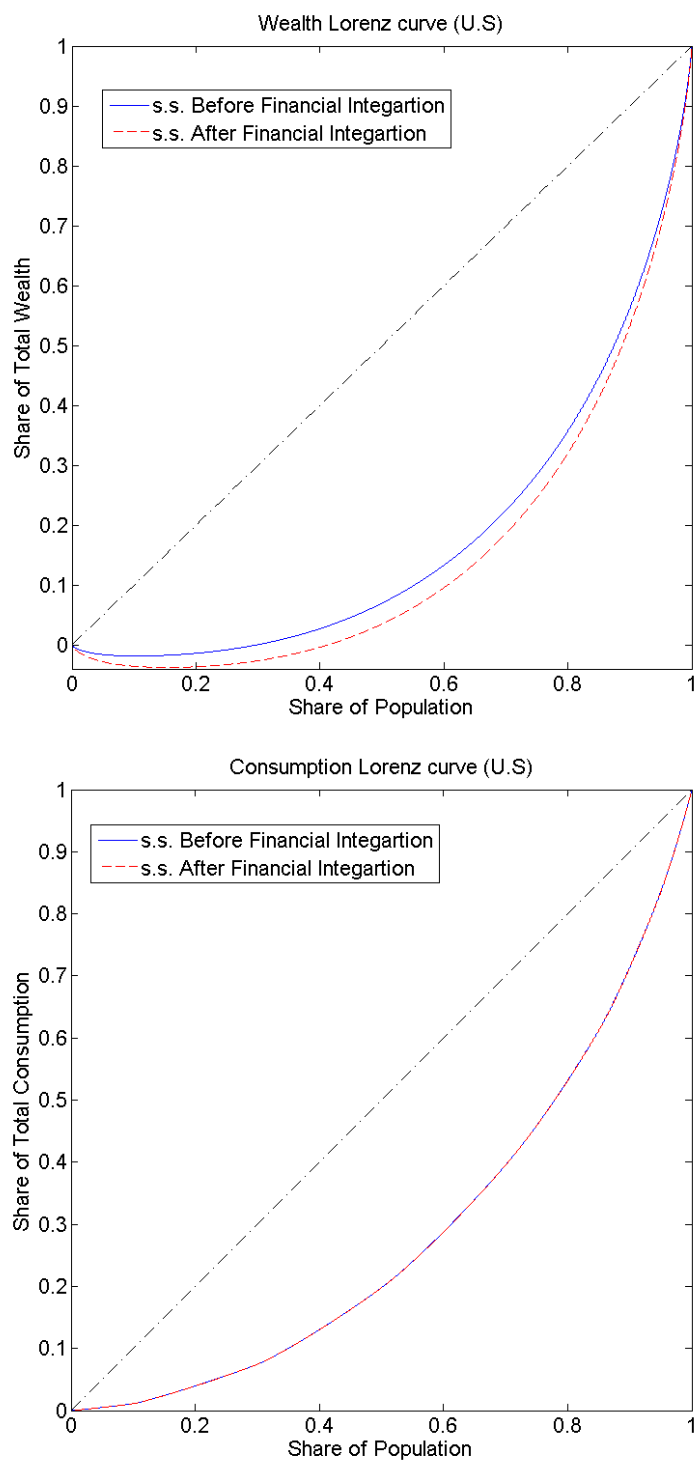


Figure 1.9: Wealth Inequality and Consumption Inequality of the U.S.

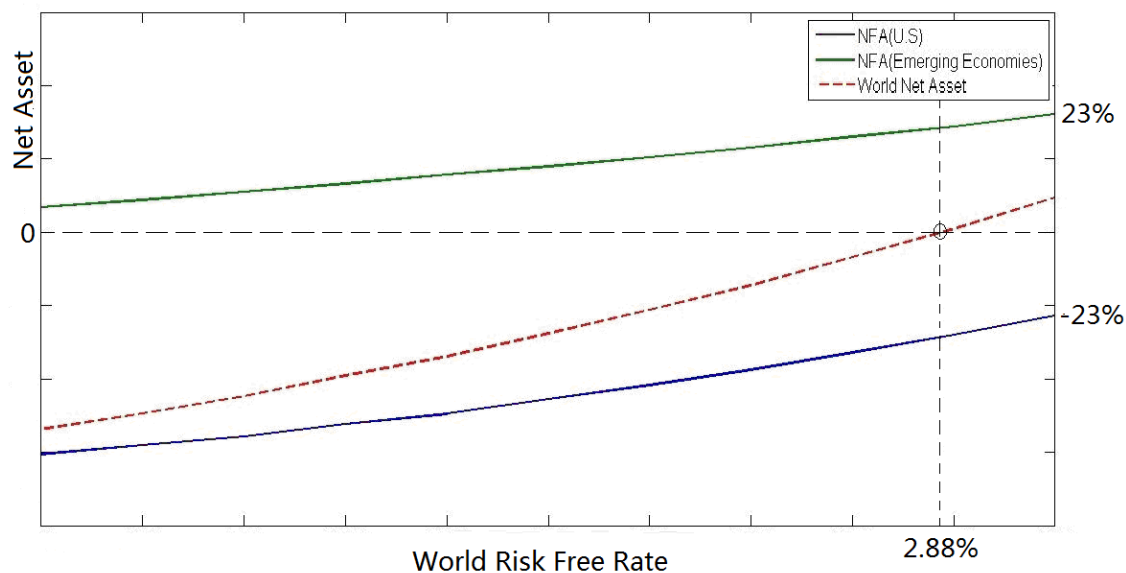


Figure 1.10: Determination of the World Risk Free Rate

1.5 Conclusion

This paper explores the long run impacts of financial integration between emerging countries and the United States. Emerging countries typically have a higher domestic loan default rate than the United States which is attributable to the less efficient domestic financial intermediation in emerging countries. In the long run, financial integration leads to global financial imbalances: emerging countries with less efficient financial intermediation and higher domestic loan default rates accumulate the U.S. assets in a gradual, long-lasting process. These patterns are consistent with the features of the global external imbalances that have been observed since the beginning of the financial integration process.

The model can generate these patterns as the natural outcome of financial integration in a stylized world in which domestic credit-monitoring efficiency

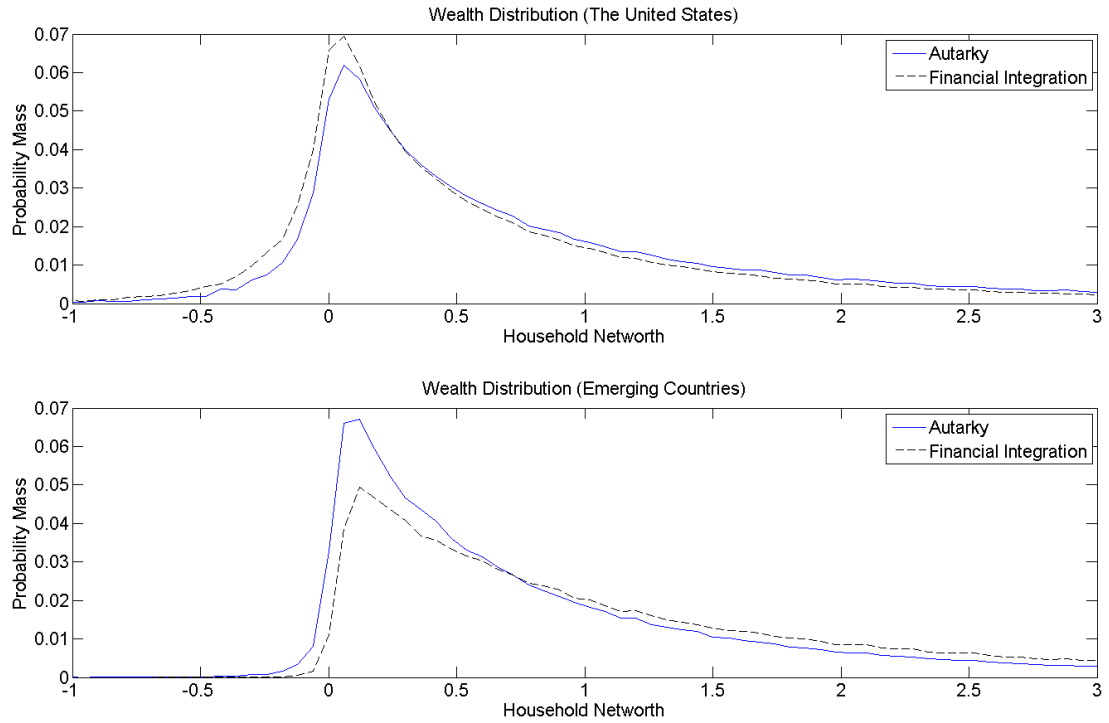


Figure 1.11: Wealth Distributions

Upper Panel - Wealth Distribution of Households in the U.S.; Lower Panel - Wealth Distribution of Emerging Country Households.

is the only source of cross-country heterogeneity. The less efficient credit monitoring that characterizes emerging countries results in higher default probability, which then disrupts the functioning of the financial sector and diminishes the aggregate credit that is available in the economy. The direct manifestations of this financial heterogeneity are a higher loan-deposit spread, higher default rates, and lower credit capacity in emerging countries compared with what occurs in the United States. All these results are consistent with stylized empirical features.

In the quantitative analysis, a two-country version of the heterogeneous-agents saving/default model is calibrated to match the data on interest rates,

income distribution, and NFA position. The autarky equilibrium suggests that, in the United States, the risk-free rate is 115 basis points higher than it is in emerging countries. As a result of the financial integration, the U.S. NFA position gradually declines to -22% of GDP and the U.S interest rate is depressed by 41 basis points.

CHAPTER 2

THE SHORT RUN RISK OF LIQUIDITY CRISIS DURING FINANCIAL INTEGRATION

2.1 Introduction

Many emerging countries such as China have been adopting gradual financial reforms to slowly integrate their financial markets with the rest of the world. One reason for this integration process being slow and gradual is that the policy makers in these countries are concerned with the potential short run risk of sudden cross-border capital flows which may bring turmoil to domestic economies.

This essay extends the modeling framework developed in the first essay to study the short run risk of capital outflow during financial integration for emerging countries. The model shows that, due to the inefficient financial intermediation, financial opening up by emerging countries might trigger a capital outflow in the short run. The sudden capital outflow from countries with less efficient financial markets raises the interest rate and crowds out domestic credit, and therefore a fraction of households in these countries become financially distressed, potentially leading to a liquidity crisis.

Given this potential risk of sudden liquidity crisis, the essay seeks policy questions pertaining whether different strategies to liberalize a country's capital account from an initial state of financial autarky would have different impacts on the economy and on welfare. To do so, several policy experiments associated with various opening-up strategies are conducted and compared.

In particular, the essay demonstrates that the aforementioned liquidity crisis

is more likely to occur if an emerging country liberalizes its capital account in an “unanticipated, once-and-for-all” fashion. In comparison, a more “gradual” opening-up strategy could help avoid the liquidity crisis, as borrowers could adjust more smoothly from a regime of cheap credit to a more expensive one. Furthermore, a short run fiscal transfer to financially distressed households in emerging countries improves welfare and can protect financial intermediaries from potential large scale default during capital account liberalization.

The analysis of this essay is built upon the modeling framework developed in essay One. The heterogeneous agents saving model with endogenous default option is well suited in accomplishing the goal of this essay. The advantages are three threefold.

First, the inclusion of the default option and endogenous debt contracts allows the model to characterize the potential “liquidity crisis” associated with a sudden capital outflow, including stylized features such as temporary spikes of the risk premium and the default rate as well as temporary credit and consumption crunches.

Second, the adoption of the two-country model enables the characterization of both the source and the destination of international capital flow. In other words, the model can compare the dynamic equilibrium of emerging countries with that of the United States during their financial integration.

The last but not the least, the incomplete-market heterogeneous-agent framework developed in essay One enables the evaluation of the welfare impact on households across wealth classes. Specifically, the model shows that in emerging economies rich households benefit from capital account liberalization

while the poor suffer. (These results are the opposite for the U.S.)

The rest of the paper is organized as follows: Section 2.2 studies the short-run dynamic equilibrium of financial integration under various opening-up scenarios, Section 2.3 discusses the welfare implications, Section 2.4 concludes.

2.2 Policy Experiments – Opening-up Scenarios and Fiscal Policies

The steady state analysis in essay One demonstrates that, before the financial integration, the market clearing risk-free interest rate is lower in an emerging country with less efficient financial market than it in the United States. Therefore, after the emerging country financially integrates with the United States, the real risk-free rate in the emerging country must go up to equalize with the U.S. interest rate. During this transition, how would households from the emerging country adjust from a cheap credit regime to one characterized by more expensive borrowing? Would different strategies to opening-up a country's capital account from an initial state of financial autarky have different impacts on the economy and on welfare?

To study these policy questions, I simulate various opening-up scenarios. As demonstrated below, households adjust their consumption/saving behavior quite differently under various opening-up scenarios. For example, a consumption crunch is more dramatic when a reform is unanticipated and once-and-for-all. I show as well that the impact of the liquidity crunch caused by financial integration can be mitigated by forming a rational expectation of regime change,

or by adopting a more gradual transitioning policy.

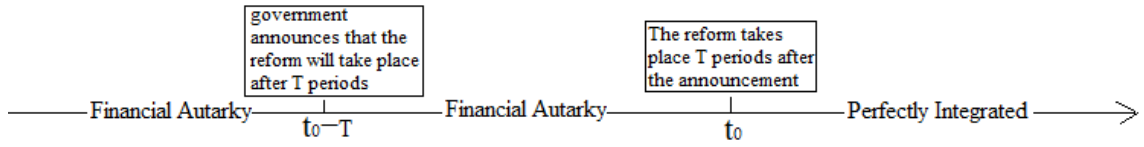
2.2.1 Models of Various Opening-up Scenarios and Fiscal Policies

Unanticipated, once-and-for-all reform

I first follow the convention of modeling financial integration as an unanticipated once-and-for-all event. This approach is embraced by many dynamic stochastic general equilibrium models of financial integration for its simplicity. Specifically, I let t_0 denote the date of the opening up, beyond which point the capital account will be completely opened to the outside world. Before t_0 , no one expects reform to occur. In this way, the opening up is treated merely as an unexpected shock to the economy.

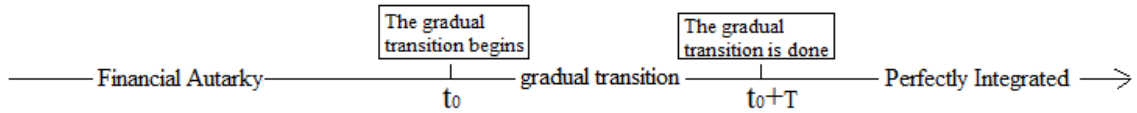
Anticipated Reform

To model an anticipated regime change, I let t_0 denote the date of the opening up, beyond which point the capital account will be completely opened to the outside world. In addition, households are perfectly informed T periods in advance that there will be a reform in period t_0 . Therefore, households have T periods to adjust their bond positions in preparing for the regime change. Obviously, when T is sufficiently large, this scenario converges on that under rational expectations, in which case there is perfect expectation of the regime change and thus nothing surprising occurs when a government makes the announcement at $t_0 - T$. Note that the forward-looking financial intermediaries in this scenario are able to adjust their bond schedules in response to the reform.



Gradual Reform with Fiscal Transfer

As in the previous cases, t_0 denotes the date of reform. However, instead of a once-and-for-all liberalization of the capital account, I let the opening-up occur gradually. In doing so, the government imposes tax rate x_0 on capital outflow.¹ In period t_0 , the government begins lowering the tax rate according to $x_t = (1 - t/T)x_0$. This tax cut happens gradually and the government takes T periods to reduce the tax rate to zero. The tax income is used to finance a lump-sum transfer $\tau_t(s_t)$ at t_0 . I let the transfer be wealth-dependent, such that it could be interpreted as a government “bailout” program when the unexpected reform occurs.



I choose one particular structure for this wealth-contingent transfer, as follows: I find $\bar{b} \in R$ and $\bar{\pi} \in R_{++}$, the transfer $\tau(b) = \pi$ if $b_0 < \bar{b}$, and $\tau(b) = 0$ otherwise. That is, households are given the lump-sum transfer if their bond positions at the beginning of period t_0 are below \bar{b} . The government’s intertemporal budget constraint is:

$$G_t + x_t B_t^{home} - \int \tau_t(b) d\mu_t^i(s, b, df) = R_{ft} G_t + 1$$

where G_t is the government’s asset holding at t , x_t is the tax rate on capital outflows, B_t^{home} is the capital outflow at period t , and $\int \tau_t(b) d\mu_t^i(s, b, df)$ represents

¹The initial tax rate x_0 is chosen to be the lowest possible value to prevent capital outflow.

the total transfer in period t ($\tau_t(b) = 0, \forall t > t_0$). Under equilibrium, the government chooses one pair $(\bar{b}, \Pi) \in R \times R_{++}$ to maintain a balanced fiscal transfer (i.e., $G_\infty = 0$.)

2.2.2 The Dynamics of Interest Rates and Endogenous Borrowing Limits

In the “unanticipated, once-and-for-all” reform scenario, the short-run dynamics of world interest rates (upper-left panel, Figure 2.1) are characterized by a sharp initial fall followed by a gradual adjustment to the new steady state. Households from less developed financial markets try to adjust in the direction of their higher target bond positions whereas households in the U.S. start to borrow more in response to the availability of cheaper capital. The reason for the interest rate overshooting is that, on the initial asset distribution, the adjustment process is more severe in emerging countries with less efficient financial intermediation (C2) relative to that of the United States (C1). The interest rate dynamic is accompanied by the evolution of the borrowing limit (upper-right panel, Figure 2.1). The endogenous borrowing limit in C2 decreases from about 60% to 52% of income to adjust for the rising interest rate and higher default rate.

In the “anticipated” reform scenario, financial integration occurs 20 periods after the government announces the news (middle-left panel, Figure 2.1). The domestic interest rate in C2 (C1) gradually declines (rises) as it gets closer to the date of reform. The movement of domestic interest rates during the pre-integration era is driven by the forward-looking behavior of households who

readjust their target bond positions in preparation for the upcoming regime change. For example, consumers with the largest debtor positions in C2 would have an incentive to adjust towards lower levels of debt as t_0 approaches. Since one household's debtor position is another household's creditor position, the domestic interest rate must drop to reduce the creditor's holdings of financial claims. Also, because this deleveraging happens in advance, the world interest rate can be equalized around the new steady-state level immediately following capital account liberalization in period 20. The evolution of the borrowing limit (middle-right panel, Figure 2.1) also reveals an interesting trend. The endogenous borrowing limit in C2 (C1) is gradually tightened (relaxed) from about 60%(90%) to 52%(101%) of income during the pre-integration era. It is somewhat counter-intuitive that the falling (rising) interest rate is accompanied by a tightening (relaxing) debt limit. This result is driven mainly by the forward-looking behavior of financial intermediaries who update the debt contracts in C2 (C1) to prepare for the rising (falling) default rate as the date of reform approaches.

In the "gradual" reform scenario, the capital tax initially creates a wedge between the two domestic interest rates, but they slowly converge on the world interest rate through a 20-year window as the capital tax is gradually reduced (bottom-left panel, Figure 2.1). The endogenous borrowing limit evolves also in a gradual fashion (bottom-right panel, Figure 2.1). The borrowing limit in C2 (C1) is gradually tightened (relaxed) during the pre-integration era.

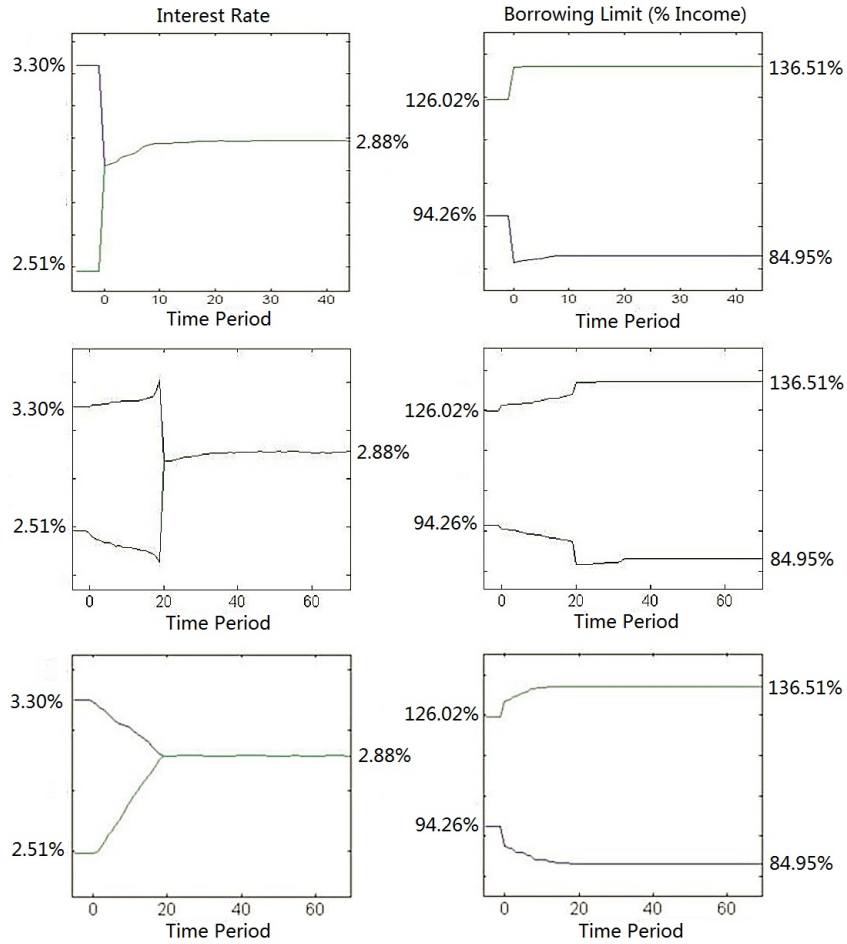


Figure 2.1: Transitory Dynamics of World Interest Rate (Left) and Endogenous Borrowing Limit (Right)

For “unanticipated, once-and-for-all” reform (Top); “anticipated, once-and-for-all” reform, (Middle); and “unanticipated but gradual” reform (Bottom). (Green line is C2 and blue line is C1)

2.2.3 A Potential Liquidity Crisis Caused by a Sudden Capital Outflow

To investigate the impact of financial integration on domestic credit markets, I look at the dynamic of the default population.

In the “unanticipated, once-and-for-all” regime-change scenario, the dynamic of the default population in C2 is characterized by a sudden spike during the reform, gradually declining toward the steady-state level (upper-left panel, Figure 2.2). The default population spikes at $t = 0$ because the reform is unexpected. Moreover, the default premium that is embedded in the bond schedule was not properly adjusted to reflect the high default rate at the time of the reform, as banks did not expect the opening up (and subsequent immediate default) at t_{-1} . As a result, some banks would fail during the reform period, subsequently leading to reducing the wealth of some of their creditors.² All banks adjust their bond schedules (risk premia) immediately following the reform, taking into account the probability of future default.

In the “anticipated” regime-change scenario, the spike in the default population during the reform period is mitigated by half because forward-looking households have already adjusted their bond positions before the opening up (middle-left panel, Figure 2.2). In this scenario, banks also foresee the opening up before t_0 and so the risk premium is properly adjusted to reflect the default rate change after the reform. As a result, banks are able to maintain zero profits throughout these transition periods.

In the “gradual” reform scenario, the default population in C2 at the period of the reform remains very low (lower-left panel, Figure 2.2). This is because the government’s fiscal transfer during that period bails out households who are under heavy debt. The fiscal transfer also protects the banks from being hurt by an otherwise unexpectedly high default rate. Moreover, the fraction

²Because this model assumes a zero-profit banking sector, the negative profit in the reform period will lead some banks to fail. Under other assumptions, these banks would be able to charge a higher risk premium in future periods to make up for their impaired balance sheets.

of the population with poor credit records remains at a low level during the transitioning periods.

Note also that the fraction of households in the United States (emerging countries) with poor credit records ends up reaching a higher (lower) steady state under all three scenarios because lower (higher) interest rate in the United States (emerging countries) induces a higher (lower) debt-to-income ratio, which in turn leads to a higher (lower) steady-state default fraction.

2.2.4 Transitory Dynamics of Current Account and Foreign Asset Positions

The dynamics of current account and foreign asset positions are similar across the three opening-up scenarios, except that the accumulation of foreign assets (liabilities) is slower under the “gradual” reform. These patterns reveal an initial current account surplus (deficit) associated with the accumulation of foreign assets by C2 (C1). Furthermore, about 12 years (20 years with gradual reform) after financial integration, this situation reverses to one in which the C2 starts to run permanent current account deficits balanced by international interest payments from the United States (Figure 2.3).

2.3 Welfare Implications

In this section I explore the welfare effects of integration for each country in various opening-up scenarios. The use of heterogeneous-household framework

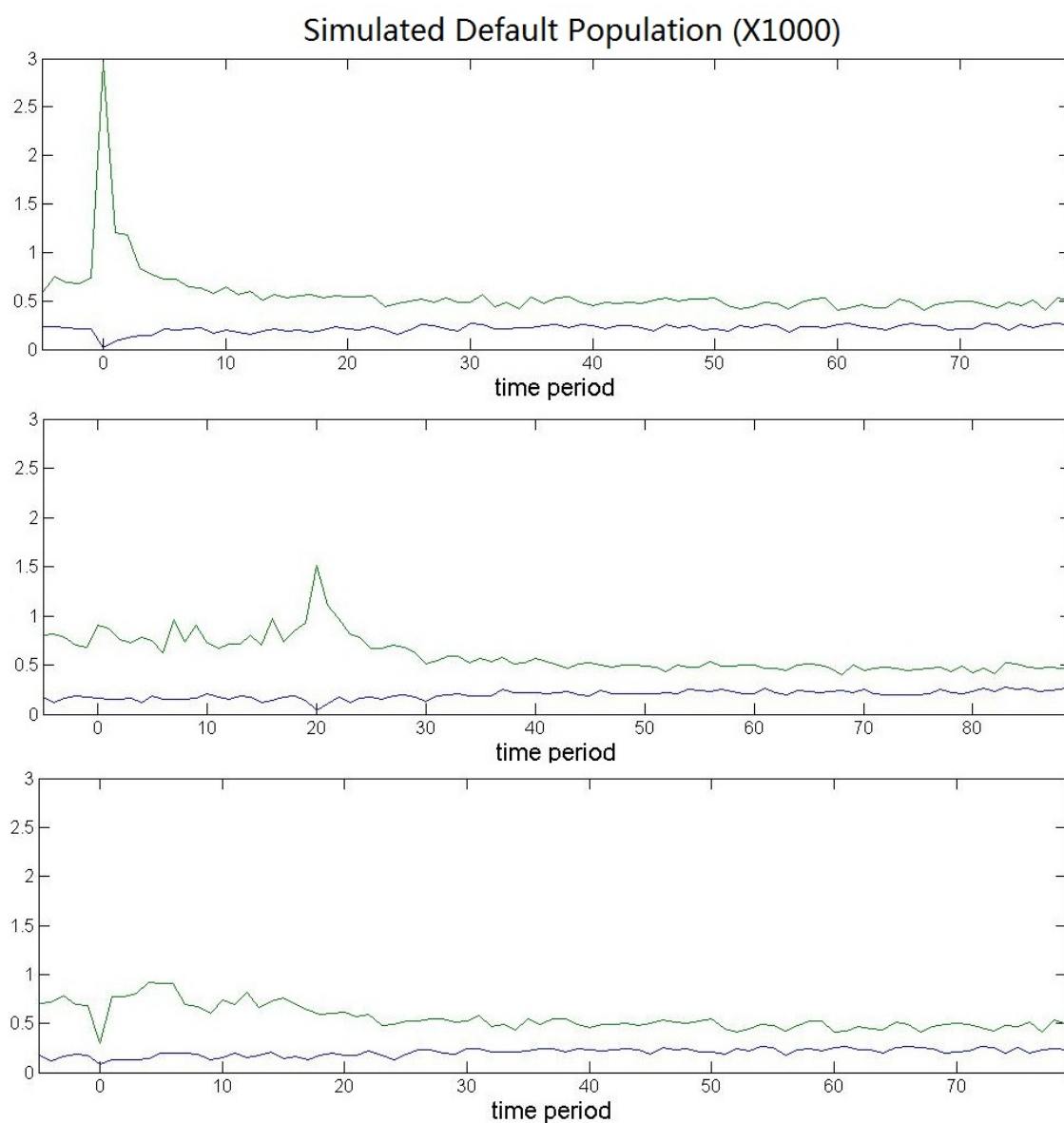


Figure 2.2: Transitory Dynamics of Default Population

“unanticipated, once-and-for-all” reform (Top); “anticipated, once-and-for-all” reform, (Middle); and “unanticipated but gradual” reform (Bottom). (Green line: C2, Blue line: C1)

allows me to distinguish how these impacts may vary between various wealth classes (i.e., poor vs. rich).

To accomplish the former, I compute a welfare measure similar to the notion of Equivalent Variation (EV) for households across wealth cohorts. Such an approach to welfare analysis is adopted in Panousi and Angeletos (2011). To accomplish the latter, I consider the aggregate utility of the whole economy (with equal weight on all households) over all transitioning periods. The whole analysis is repeated for different opening-up scenarios and fiscal policies.

2.3.1 Welfare Impact for Various Wealth Classes:

Quantitatively, for a country j , I focus on households whose realization of productivity is in the “middle state”, letting $V_{\infty, aut}^i(b)$ denote the value functions in the autarkic steady state, and letting $V_{0, int}^i(b)$ denote the value functions at the beginning of the reform period, respectively. The task is to compute, for each level, the bond position b and a compensating transfer $e_j(b)$ such that $V_{\infty, aut}^i(b + e_j(b)) = V_{0, int}^i(b)$. The interpretation of $e_j(b)$ runs as follows: suppose that an emerging country that is currently in financial autarky contemplates the option of liberalizing its capital accounts to enable it to integrate financially with the United States. For a particular bond position b , $e_j(b)$ is the minimal compensation that a household with bond position b would agree to accept in return for the cancellation of such a reform. Therefore, $e_j(b)$ is closely related to the notion of Equivalent Variation.³

For this exercise, I then express the corresponding compensating differential as a fraction of the household’s annual income. The resulting number represents

³Equivalent Variation is often interpreted as the change in a consumer’s wealth that would be equivalent to a price change in terms of its welfare impact. See Mas-Colell, Whinston and Green (1995).

a welfare gain if it is positive and a welfare loss if it is negative. These welfare gains and losses for various wealth cohorts are illustrated in Figure 2.4 for each of the two countries and three opening-up scenarios.

I first consider the benchmark scenario of “anticipated once-and-for-all” reform (blue lines) for C2 (two panels on the right). In general, financial integration benefits the rich at the expense of the poor: the poor suffer losses, whereas the rich enjoy gains. The intuition is the following. Financial integration raises interest rates in C2, leading to a decline in the present discounted value of future household labor income. This hurts all households, but the adverse effect is stronger on the poorer ones, since consumption in poorer households depends more on labor income. Meanwhile, the higher interest rate implies that both returns on savings and the cost of borrowing have increased. These effects benefit the rich at the expense of the poor as rich households are creditors while poor households are debtors. For extremely rich households who are in the right tail of the wealth distribution, the positive effect is strong enough to offset the negative effect, so these households benefit from financial integration. In addition, the extreme poor in the left tail of the wealth distribution are those who suffer the most, because declining credit capacity forces these households to undergo a deleveraging process, which results in a short-run consumption crunch.

For C1 (two panels on the left), the situation is quite the opposite. The poor and the middle class gain, while the very rich lose. The intuition for these results is analogous to that for C2. C1’s poor gain immediately upon integration because of the increase in human wealth and relaxed borrowing limits, while the rich lose because of lower returns on their bond holdings. Moreover, it can also be observed that the abovementioned welfare effects are mitigated in the

Table 2.1: Aggregate Equivalent Variation

	Unanticipated	Anticipated	Gradual
C1	0.0435	0.0041	0.0148
C2	-0.0321 (-0.0364)	-0.0071	-0.022 (-0.0219)

other two opening-up scenarios.

Accounting for the asset distribution, I can compute the aggregate equivalent variation for the whole economy. Table 2.1 summarizes the aggregate welfare gains and losses across all households for each country. These numbers show that, in general, C1 benefits from financial integration while C2 suffers. Moreover, the social welfare lost in C2 could be reduced by adopting a more gradual and anticipated opening-up strategy.

2.4 Conclusion

This essay studies a potential credit-crunch event stemming from sudden capital outflow during financial integration between emerging countries and the United States.

In the short run, a sudden, once-and-for-all financial integration may potentially result in a credit crunch in emerging countries: such capital market integration will raise the interest rates in these countries and suppress the endogenous borrowing limit; thus, a fraction of agents will become financially distressed, subsequently generating a temporarily high risk premium, a high default rate, and a consumer-spending crunch.

This paper provides not only a possible explanation of the abovementioned stylized facts, but also important policy lessons pertaining to how an emerging economy should liberalize its capital accounts from an initial state of financial autarky. To investigate this issue, the paper conducts several policy experiments associated with various opening-up scenarios.

The paper first follows the convention of modeling the financial integration as an “unanticipated, once-and-for-all” event. This is embraced for its simplicity by most dynamic stochastic general equilibrium models for financial integration. I then ask: what if the government perfectly informs households in advance that the country’s capital account will be liberalized during certain periods. Such an “anticipated” opening-up scenario is similar to the case under rational expectations, in the sense that agents and financial intermediaries perfectly foresee the future regime change and respond accordingly. The third policy experiment corresponds to a “gradual” opening-up scenario. Using this “gradual” strategy, the government imposes a tax on capital flows. Initially the tax rate is sufficiently high to preclude international capital mobility. After the reform, the government gradually reduces the tax rate. In addition, the government uses the tax income to provide a bailout program by means of a wealth-contingent wealth transfer.

The paper shows that consumption crunches and defaults are significantly stronger under “unanticipated, once-and-for-all” reforms, compared with those under the “anticipated” and “gradual” reform strategies. Furthermore, “anticipated” and “gradual” reform protects domestic financial intermediaries from being hurt by a large-scale credit default.

Using a framework consisting of a large number of heterogeneous house-

holds allows the model to track the change in wealth distribution and consumption distribution in each country affected by the financial integration. Following financial integration, wealth distribution becomes more dispersed in the United States as well as the emerging countries, implying growing wealth inequality. Moreover, the wealth distribution gradually shifts to the right for households in emerging countries whereas for households in the U.S. it shifts to the left. In the short run, the consumption inequality in the United States is mitigated due to improved credit conditions following financial integration. The analysis of Equivalent Variations provides the welfare implications of financial integration across various wealth levels as well as at the aggregate level: the benefit of financial integration is higher (lower) for wealthy households relative to the poor in emerging economies (the U.S). These welfare impacts are strongly linked with the shape of a country's wealth distribution. At aggregate level, emerging economies (the United States) may suffer (benefit) from financial integration. In addition, the paper shows that gradual change in financial openness mitigates these differences, leading to higher overall welfare. Accordingly, the paper argues for a more gradual approach to capital account opening for emerging countries.

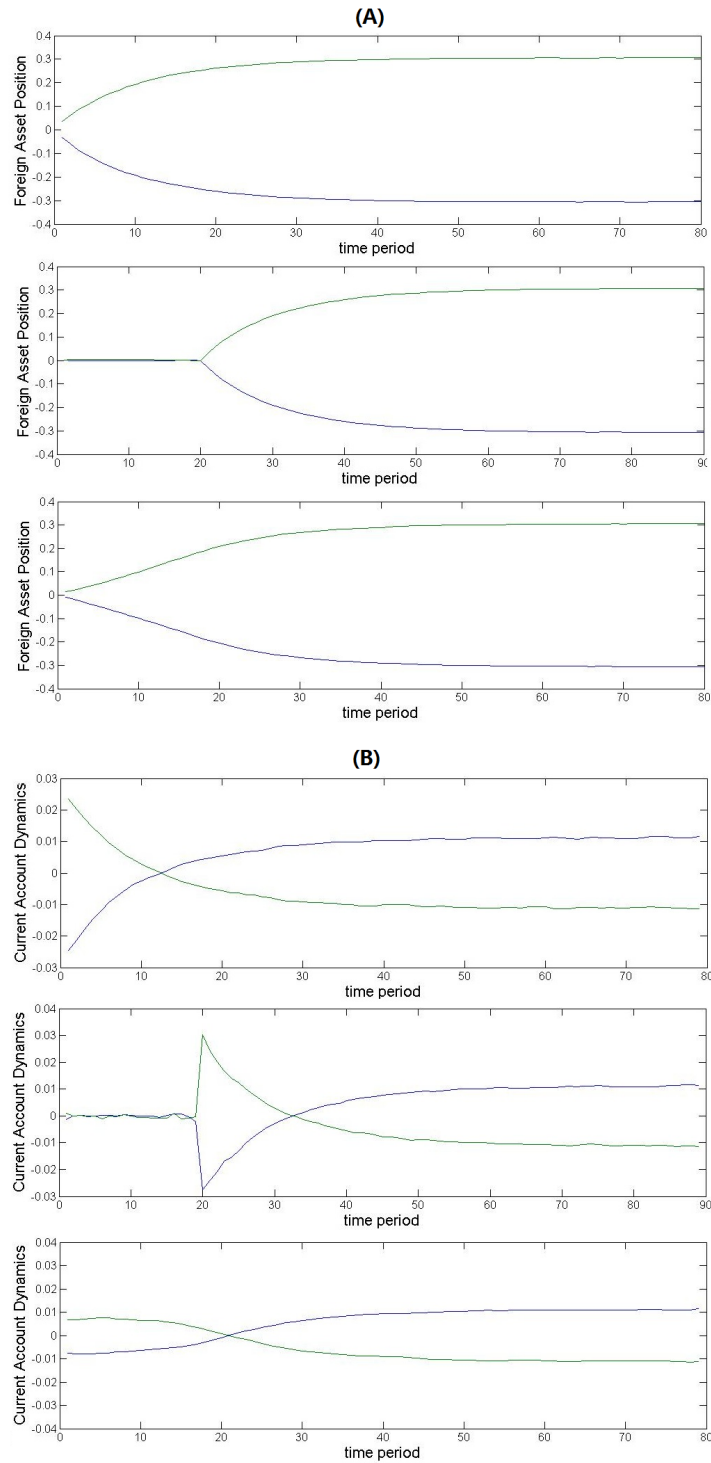


Figure 2.3: Transitory Dynamics of Foreign Asset Position (A) and Current Account (B)

“unanticipated, once-and-for-all” reform (Top); “anticipated, once-and-for-all” reform, (Middle); and “unanticipated but gradual” reform (Bottom). (Green line: C2, Blue line: C1)

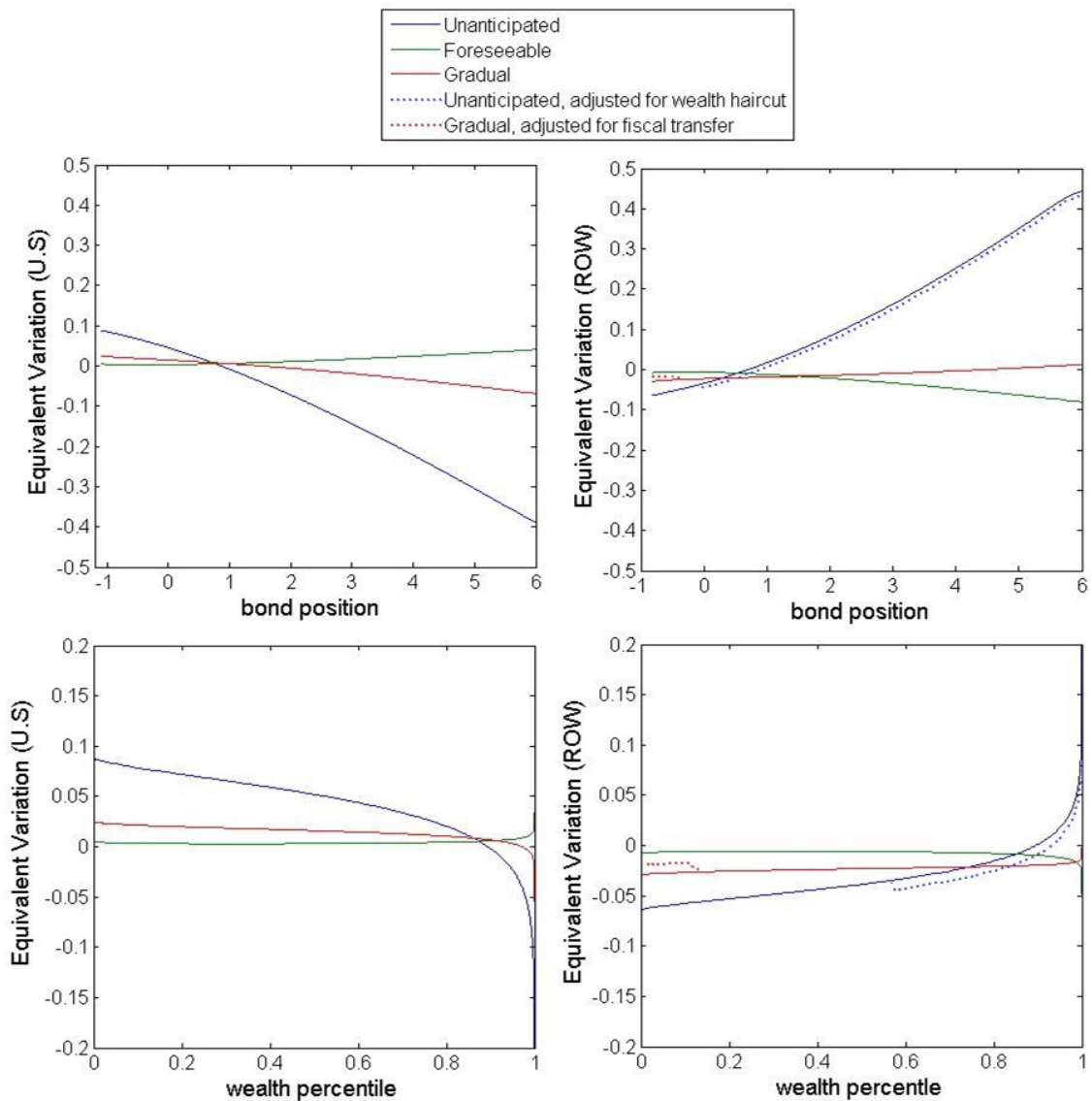


Figure 2.4: Welfare Implication

The blue line in this figure represents the welfare effects in the first opening-up scenario (i.e. unanticipated, once-and-for-all reform), the green line represents the welfare effects in the second opening-up scenario (i.e. anticipated, once-and-for-all reform), the red line represents effects in the third opening-up scenario (unanticipated but gradual transition with fiscal policy). The dotted blue line takes into account the wealth hair cut of depositors during unanticipated reform. The dotted red line takes into account the fiscal transfer ("bail-out") during the gradual reform

CHAPTER 3

THE IMPACT OF FINANCIAL DISRUPTIONS ON BUSINESS CYCLES

3.1 Introduction

The financial market is important insofar as it helps allocate funds to the most productive investors. Financial market trouble interferes with efficient allocation of resources, leading to a significant decline in output. In 2008, virtually all advanced economies and many emerging markets experienced deep recessions. A common feature of these recessions was that they were accompanied by credit crunches coinciding with a sharp declines in capital prices, a type of episode commonly referred to as “financial disruption” (Claessens, Kose and Terrones, 2011). The need to understand such unprecedented economic contractions and concurrent financial market turmoil that have occurred since World War II motivated me to develop a stylized macroeconomic model that highlights the intrinsic linkage between financial disruptions and business cycles.

In retrospect, we can see that episodes of financial disruption differ markedly from typical business cycles (Mendoza 2010). For example, the volatility of business cycle fluctuation had been low in the United States from the mid-1980s into the first decade of the 2000s. In sharp contrast, this era of “great moderation” ended with a severe economic contraction associated with financial disruption, whereby the U.S. economy experienced highly synchronized declines in output, credit, and the value of capital as well as spikes in volatility and the risk premium. The amplitude of these fluctuations ran far beyond the scale of normal business cycles. Moreover, a rich body of empirical literature that has reported evidence from a wide range of countries suggests that

recessions associated with financial disruptions tend to be significantly longer and deeper than other recessions (Reinhart and Rogoff 2008, Claessens, Kose and Terrones, 2011). These findings suggest that financial disruptions should be modeled as rare, distinct episodes in business cycles during which the system behaves differently from how it behaves in normal cycles.

Moreover, financial disruptions seem to be recurring scenarios marked by certain empirical regularities. Reinhart and Rogoff (2008) argue that the 2008 financial disruption was qualitatively and quantitatively comparable to 18 earlier post-war financial crises in industrialized countries. For example, they found a common slowing of economic growth on the eves of such episodes (Reinhart and Rogoff, 2008). Based on historical data taken from 48 countries, Mendoza and Terrones (2008) find that most episodes of financial disruption in emerging markets were associated with credit booms. All these regularities suggest that financial disruptions might not happen completely randomly. Therefore, to understand the intrinsic linkage between financial disruptions and business cycles, it is important to explore the nature of these events: to what extent are financial disruptions exogenous and to what extent do they simply evolve as outcomes of dynamic equilibrium?

To address these issues, this paper develops a stylized macroeconomic model with credit frictions. In order to characterize rare episodes of extraordinary financial disruption and understand their intrinsic linkages with business cycles, the model I propose incorporates two distinct regimes, “normal cycles” and occasional episodes of “financial disruption,” and endogenizes the switch between these two regimes, so that financial disruptions are rare, endogenous episodes evolving over business cycles.

Several studies have made serious efforts to explain how financial frictions amplify shocks. For example, Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999) argue that, due to financial frictions and the resulting constraints on borrowing, the reduced net worth of levered agents leads to a drop in prices for their assets, further lowering their net worth, in a vicious circle.

The recent global economic crisis returned this issue to the forefront of economic concerns. Since then, a new stream in the macro literature on endogenous risk and periodically binding financial constraints has begun to emerge. For example, Mendoza (2010) uses a business cycle model with a collateral constraint to explain financial crashes and subsequent deep recessions for a small, open, emerging economy. The model shows that precautionary savings limit collateral constraint binding to occasional occurrences followed by sudden stops. The paper highlights the importance of using nonlinear global methods to characterize periodically binding credit constraints. Brunnermeier and Sannikov (2013) develop a continuous time model in which they do not assume that after a shock the economy drifts back to the steady state, instead allowing the length of the slump to remain uncertain.

My paper builds on the aforementioned literature. Like Kiyotaki and Moore (1997) and Brunnermeier and Sannikov (2013), I assume that there exist two types of agents, high-productivity agents and low-productivity agents, distinguished by their contrasting ability to produce output using capital and labor. In this paper all agents are rational, forward-looking, and risk averse. Agents can default on their liabilities at any time, so bonds are traded only if bond payments are backed by collateral. Capital serves as a factor of production and as

collateral for loans. Both bonds and capital are traded in the financial market.

Since the return on capital is higher if it is owned by high-productivity agents, in equilibrium high-productivity agents accumulate capital stocks through leverage. Due to the debt enforcement problem, there is a maximum level of leverage determined by the financial market, which depends on the market's projection of the future value of collateral. The equilibrium leverage of high-productivity agents occasionally hits the endogenous maximum level, in which case they become financially constrained. Because of the precautionary motive, there is only a low probability that the leverage constraint binds, while the absence of constraint characterizes the economy most of the time. Thus, the model is able to incorporate two distinct regimes: "normal cycles" and rare episodes of "financial disruption."

During normal cycles high-productivity agents are not financially constrained, and they are thereby able to maintain an optimal level of capital stock through collateral borrowing. Therefore, during normal business cycles disturbances in the system remain near the center of the stochastic steady state distribution whereby the economy is having an efficient capital allocation. In other words, during normal cycles shocks are not amplified by the financial market.

During episodes of financial disruption, high-productivity borrowers are financially constrained, in which case they must sell capital to low-productivity agents, leading to lower capital prices. The decline in capital prices generates an adverse feedback loop by shrinking financial wealth, in particular the value of collateralizable capital, which further suppresses borrowing capacity in the presence of credit frictions. Meanwhile, financial disruptions have a strong negative impact on business cycles as they result in misallocation of capital to low-

productivity agents, leading to a loss of efficiency in production. This explains why economic contractions associated with financial disruptions are stronger than typical recessions during normal cycles.

Due to this highly nonlinear spiral effect, financial disruptions have unique features such as Fisher debt deflation, plunging output and capital prices, and rising risk-premia and Sharpe Ratios associated with capital, all of which are significantly different from market conditions in normal cycles. Moreover, because it takes time for levered agents to rebuild their wealth, recovery from a financial disruption might be slow, and in these circumstances the scale of the slump is uncertain.

The switch between the two regimes is endogenous, the probability of which gradually changes with equilibrium leverage. Specifically, when the leverage ratio of high-productivity agents is far below the endogenous maximum level, the probability of financial disruption is relatively low; when high-productivity agents' leverage ratio approaches the maximum level, the probability of financial disruption is high, as even a small shock could force the leverage constraint to bind. The evolution of equilibrium leverage depends on the history of macro shocks that affect all agents' balance sheets. Thus, rare episodes of financial disruptions are not caused entirely by exogenous shocks. Instead, financial disruptions are equilibrium outcomes evolving with the state of the economy.

In order for the system to switch endogenously between two distinct regimes, the model must be solved globally with multiple periodically binding constraints. I solve for the full dynamics of the model using a projection method similar to that of Kubler and Schmedders (2003). The solution is characterized within the entire state-space as a stationary Markov equilibrium, and

the modeling results are presented in a reasonably calibrated example.

The remainder of the paper is organized as follows: Section 3.2 briefly reviews the relevant literature. Section 3.3 introduces the model and characterizes the solution; Section 3.4 discusses the quantitative properties of the model; Section 3.5 highlights some key model features and presents the simulated results; Section 3.6 concludes.

3.2 Literature Review

There is a substantial body of macro literature focusing on the effects of financial friction on the macro economy. Early studies explored various financial friction mechanisms in stylized setting. For example, Carlstrom and Fuerst (1997) show that endogenous agency costs can potentially alter business cycle dynamics. Holmstrom and Tirole (1997) use information asymmetry to explain how the distribution of wealth across firms, intermediaries, and investors affects economic activity. Kiyotaki and Moore (1997) introduce the debt enforcement problem and study how the price of collateral assets interacts with business cycles.

A comprehensive framework linking financial friction with real activities is developed by Bernanke, Gertler, and Gilchrist (1999), whose study can be seen as a synthesis of these earlier studies on financial frictions, incorporating some of their New-Keynesian features into the model in order to enhance its empirical relevance. The BGG framework is widely adopted in studies of numerous relevant topics such as exchange rate policy (Céspedes, Chang, and Velasco 2004 and Gertler, Gilchrist, and Natalucci 2007), banking and monetary policy (Gerali, Neri, Sessa and Signoretti 2010), and housing values (Aoki, Proudman and

Vlieghe 2004; Iacoviello 2005).

Following the most recent financial crisis, studies emphasizing its non-linear effect on the economy with periodically binding financial constraints began to emerge, including the aforementioned Mendoza (2010) and Brunnermeier and Sannikov (2013). Moreover, Heathcote and Perri (2012) document a systematic negative relationship between wealth and volatility. Perri and Quadrini (2013) introduce a two-country model with credit shocks appearing as the results of a self-fulfilling equilibrium. By solving the model in the global state-space, they are able to endogenize the severity of economic contractions.

On the other hand, there is also a substantial body of finance literature focusing on financial market equilibrium and the impact of market frictions on the dynamics of financial variables, namely asset prices, leverage, and the credit supply. For example, Stiglitz and Weiss (1981) use moral hazard and adverse selection under asymmetric information to explain why credit may be rationed in equilibrium. Shleifer and Vishny (1992) argue that the liquidation value of assets may be low in poor economic times, accounting for the private cost of leverage, which can explain the variation in debt capacity. Geanakoplos (2010) assumes heterogeneous beliefs to show that equilibrium determines not only interest rates but also leverage, and that leverage cycles could lead to fluctuations in asset prices. Araujo, Kubler, and Schommer (2010) and Brumm, Grill, Kubler, and Schmedders (2013) develop models with equilibrium default, the former investigating the effect of collateral requirements on risk sharing with the latter examining the collateral value of long-lived assets. Rampini and Viswanathan (2010) study collateral equilibrium from the view of corporate finance. They show that more productive firms may be more likely to exhaust their debt ca-

capacity and that capital may be less efficiently deployed in downturns. He and Krishnamurthy (2012, 2013) develop a continuous-time model focusing on the role of financial intermediaries on asset pricing. The former study generates observed patterns during financial crises, including the Sharpe Ratio, volatility, the correlation of asset prices, and interest rates. The latter replicates the dynamics of risk premia during crises.¹

My paper fits into both of the abovementioned streams of the literature by highlighting the macro-financial linkage through collateral borrowing. I show that when the leverage ratio of an economy is high, there exists an endogenous vicious circle that could potentially synchronize economic contractions and disruptions of financial markets.² The financial market foresees the future risk, and therefore asset prices and collateral requirements all change with the equilibrium leverage.

Last but not least, Kubler and Schmedders (2003) provide a theoretical foundation for my paper. Building upon the work of Duffie, Geanakoplos, Mas-Colell, and McLennan (1994) and Geanakoplos and Zame (2002), Kubler and Schmedders assume the existence condition of stationary Markov equilibria for a dynamic pure exchange economy with incomplete markets and collateral constraints. They show that such stationary Markov equilibria can be characterized

¹Many other papers belonging to these two strands are also relevant but cannot be discussed here in detail. See Goldstein and Razin (2013) for an extensive review of theories of financial crisis, and see Brunnermeir, Eisenbach, and Sannikov (2012) for a comprehensive survey of the literature on macroeconomics with financial frictions.

²In other words, financial disruptions and the associated economic contractions are not necessarily caused by large, disastrous shocks. When the equilibrium leverage approaches to its endogenous maximum level, even small exogenous shocks can occasionally trigger large contractions.

by a mapping from exogenous shocks and the current distribution of financial wealth to prices and portfolio choices. I extend this equilibrium concept to a production economy.

3.3 The Model

3.3.1 Basic Environment

(1). Two Types of Agents: There are two types of agents ($h \in \{1, 2\}$) in the proposed model, distinguished by their unequal ability to use capital productively high-productivity agents measured by μ_1 and low-productivity agents measured by μ_2 . The total population of the economy is normalized to one ($\mu_1 + \mu_2 = 1$). Following Brunnermeier and Sannikov (2013) as well as Perri and Quadrini (2012), I also assume that high-productivity agents discount the future more than low-productivity agents. The difference in the discounting factor β_h implies the equilibrium leverage, as revealed in the data.³ In the Appendix, I verify that without the difference in β_h , the underlying mechanism of the model remains the same.

(2). Preference: Both types of agents have standard CRRA utility. $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$

(3). Technology: Both types of agents produce according to Cobb-Douglas technology:

$$\theta_h(s_t)k_h^\phi l_h^{1-\phi}$$

³One alternative approach would be to model the tax benefit on external borrowing. The assumption of heterogeneous discounting factors is more intuitive and avoids unnecessary distortion of the economy.

where k_h is the capital owned by a type- h agent and l_h is the labor hired by a type- h agent. θ_h is the type-specific stochastic process for productivity. $\lambda\theta_1(s') = \theta_2(s')$, with $\lambda < 1, \forall s'$, that is, type 1 (the expert) being more productive than type 2 at all nodes of the history.

- (4). **Capital k :** There is a fixed amount that equals one unit of capital k that can be used as collateral. The unit of capital is traded in every period so that its allocation $\{k_1, k_2\}$ and price q^k are determined in equilibrium.
- (5). **Bond b :** A bond is a one-period security with zero-net supply. It promises one unit of consumption good at each period. Agents can default on their bond liabilities at any time; therefore, bonds must be backed by capital k . The equilibrium collateral requirement κ varies across nodes of the history, as the market anticipates changes in the future value of collateral.
- (6). **Uncertainty:** The only exogenous shocks are productivity shocks. At each period $t \geq 0$ one of M possible exogenous shocks $s_t \in S = \{1, \dots, M\}$ occurs. One history path up to period t is denoted by $s^t \in S^T$, which corresponds to one node on the entire event tree S^T . The productivity process follows an AR1 process with persistence ρ and standard deviation δ .⁴

Assuming no individual-level risk, or perfect risk sharing within each type, the aggregation can be taken within each type. Thus, I can study a representative household for each type.

⁴In the numerical simulation, the AR1 process is approximated by a first-order Markov transition matrix with M states.

3.3.2 Budget Constraint

A type- h agent starts the period with financial net worth $a_{h,t}$, which equals the current market value of the agent's existing capital stocks and bond holdings (i.e., $a_{h,t} = q_{h,t}^k k_{h,t-1} + b_{h,t}$). After observing productivity $\theta_h(s^t)$ for the period, the agent decides how much capital $K_{h,t}$ to invest in production. Similar to Perri and Quadrini (2009) and Kubler and Schmedders (2003), I assume intra-period production, that is, capital input allows the agent to generate output in the same period.⁵ The budget constraint is

$$q_{h,t}^b b_{h,t+1} = a_{h,t} + w_t e_{h,t} - c_{h,t} - q_{h,t}^k k_{h,t} + F_h(k_{h,t}, l_{h,t}, \theta_h(s^t)) - w_t l_{h,t} \quad (3.1)$$

where $a_{h,t}$ is the beginning period net worth, $w_t e_{h,t}$ is wage income (the market equilibrium wage multiplied by the individual labor endowment), $F_h(k_{h,t}, l_{h,t}, \theta_h(s^t)) - w_t l_{h,t}$ equals profits from production (which is production revenue $F_h(k_{h,t}, l_{h,t}, \theta_h(s^t))$ minus labor wage payments $w_t l_{h,t}$). After plugging in net worth $a_{h,t} = q_{h,t}^k k_{h,t-1} + b_{h,t}$, and using $i_{h,t} = q_{h,t}^k (k_{h,t} - k_{h,t-1})$ to denote the net capital investment, the budget constraint can be re-written as:

$$c_{h,t} + i_{h,t} - (F_h(k_{h,t}, l_{h,t}, \theta_h(s^t)) - w_t l_{h,t}) - w_t e_{h,t} = b_{h,t} - q_{h,t}^b b_{h,t+1}; \quad (3.2)$$

that is, consumption $c_{h,t}$ plus net capital investment $i_{h,t}$ minus production profit $F_h(k_{h,t}, l_{h,t}, \theta_h(s^t)) - w_t l_{h,t}$ and wage income $w_t e_{h,t}$ equals current bond payments $b_{h,t}$ net of new bond purchases $q_{h,t}^b b_{h,t+1}$.⁶

⁵The assumption of intra-period production helps reduce the number of state variables. The use of inter-period production would not change the key properties of the model but it would complicate the numerical solution by enlarging the state-space.

⁶Suppose the production input expenditure occurs before investment revenue materializes; the agent could then potentially face a cash-flow mismatch within the period. One interpreta-

3.3.3 Debt Contracts

When discussing debt contracts, it is convenient to denote the positive bond position and negative bond position separately, as in $b_{h,t(+)} = \max(0, b_{h,t})$, and $b_{h,t(-)} = -\min(0, b_{h,t})$, and thus $b_{h,t} = b_{h,t(+)} - b_{h,t(-)}$. Moreover, the (net) risk-free interest rate is: $r_t = 1/q_t^b - 1$.

The inter-temporal debt contracts are not perfectly enforceable because the agent can default. Default can take place at any time. Therefore, debt repayment must be guaranteed by using capital as collateral. Obviously not all capital is collateralizable. Typically, some capital is super liquid, which can be easily diverted in case of default. I use a straightforward approach by assuming that an agent with k units of productive capital can use ϵk as collateral. In other words, when an agent defaults he can easily divert $1 - \epsilon$ fraction of his productive capital in case of default, and banks confiscate all the collateral, which is the remaining ϵ share of the total productive capital.

Default gives the lender the right to liquidate the agent's assets. After such a diversion, however, the only remaining asset is the collateral $\epsilon k_{h,t}$. Suppose that the liquidation value of the capital is $\epsilon k_{h,t} q_{h,t+1}^k$. The borrower defaults whenever the market value of the collateral is lower than the value of the debt, that is: $b_{h,t(-)} < \epsilon k_{h,t} q_{h,t}^k$. To ensure that borrowers do not default, the total liabilities at any time must be subject to the following enforcement constraint:

$$b_{h,t+1(-)} \leq \epsilon k_{h,t+1} \min(q_{h,t+1}^k); \quad (3.3)$$

that is, the value of the debt must be less than the value of the collateral across

tion is that the agent can cover the cash mismatch by issuing risk-free intra-period loans. The production output is used as collateral so that the loan is always repaid after the revenue begins to accrue. As the loan is risk-free and within the period, there is no interest.

all future scenarios.

Following Brunn et al. (2011), the collateral requirement is defined as the unit of capital required as collateral in order for borrowers to sell one bond. Then, from equation (3), it is easy to see that the minimum collateral requirement $\kappa_t(s_t)$ to guarantee a default-free state is:

$$\kappa_t(s_t) = \frac{1}{\min_{s_{t+1}}(q_{t+1}^k)}, \quad (3.4)$$

and so equation (3) can be re-written as:

$$b_{h,t+1(-)}\kappa_t(s_t) \leq \epsilon k_{h,t+1}; \quad (3.5)$$

that is, if an agent issues one bond, he is required to put up κ units of capital as collateral.

3.3.4 Competitive Equilibrium

Under competitive equilibrium, each individual agent takes as given the equilibrium capital price $\{q_t^k(s^t)\}_{s^t \in S^T}$, the bond price (and so the interest rate) $\{q_t^b(s^t)\}_{s^t \in S^T}$, and the unit labor wage $\{w_t(s^t)\}_{s^t \in S^T}$. A type- h agent optimizes his expected discounted utility by choosing paths of consumption $c_h = \{c_{h,t}(s^t)\}_{s^t \in S^T}$, capital stocks $k_h = \{k_{h,t}(s^t)\}_{s^t \in S^T}$, bond positions $b_h = \{b_{h,t}(s^t)\}_{s^t \in S^T}$ and labor hiring $l_h = \{l_{h,t}(s^t)\}_{s^t \in S^T}$. The optimization problem is specified below:

$$\max_{\{c_h\}, \{k_h\}, \{l_h\} \geq 0, \{b_h\}} E\left\{ \sum_{t=0}^{\infty} \beta_h^t u(c_{h,t}(s^t)) \right\} \quad (3.6)$$

subject to $\forall s^t \in S^T$

$$a_{h,t}(s^t) = b_{h,t}(s^t) + q^k(s^t)k_{h,t}(s^t) \quad (3.7)$$

$$\begin{aligned}
& c_{h,t}(s^t) + q^k(s^t)k_{h,t+1}(s^t) + q^b(s^t)b_{h,t+1}(s^t) \\
& = a_{h,t}(s^t) + (F_h(k_{h,t+1}, l_{h,t}, \theta_h(s^t)) - w_t(s^t)l_{h,t}(s^t)) + w_t(s^t)e_{h,t}(s^t)
\end{aligned} \tag{3.8}$$

$$b_{h,t+1}(-)(s^t) \leq \epsilon k_{h,t+1}(s^t) \min_{s_{t+1}}(q_{t+1}^k(s^{t+1})), \tag{3.9}$$

where equation (7) and (8) represent the inter-temporal budget constraints and equation (9) represents the borrowing limit. The paths of prices $\{q_t^k(s^t), q_t^b(s^t), w_t(s^t)\}_{s^t \in S^T}$ are simultaneously determined in equilibrium. Formally, competitive equilibrium is defined as follows:

Definition 3 . Competitive Equilibrium:

Competitive equilibrium for the above-described economy with initial allocation of capital $(k_{h,0})_{h \in 1,2}$ and initial shock s_0 is a collection

$$\{(c_{1,t}(s^t), b_{1,t}(s^t), k_{1,t}(s^t), l_{1,t}(s^t)), (c_{2,t}(s^t), b_{2,t}(s^t), k_{2,t}(s^t), l_{2,t}(s^t)), q_t^k(s^t), q_t^b(s^t), w_t(s^t)\}_{s^t \in S^T}$$

satisfying the following conditions:

- (1). *For each agent h , the path $(c_{h,t}(s^t), b_{h,t}(s^t), k_{h,t}(s^t), l_{h,t}(s^t))_{s^t \in S^T}$ solves the agent's utility maximization problem defined by equations (6) through (9).*
- (2). *The bond market clears: $\sum_h \mu_h b_{h,t}(s^t) = 0$*
- (3). *The capital market clears: $\sum_h \mu_h k_{h,t}(s^t) = 1$*
- (4). *The labor market clears: $\sum_h \mu_h l_{h,t}(s^t) = \sum_h \mu_h e_{h,t}(s^t)$*

The existence proof given in Kubler and Schmedders (2003) can be generalized to show that there exists a Markov equilibrium equivalent to the competitive equilibrium defined above. Furthermore, it can be shown that all aggregate

variables (e.g., w, q^k, q^b, y, Ω_i) and policy correspondences (e.g., $c^h(:, :), b^h(:, :), k^h(:, :), l^h(:, :)$) depend on the distribution of wealth in the economy as well as the exogenous state. In the environment in which my model operates, the endogenous state-space can be reduced to a single variable – the wealth share of the high-productivity agent:

$$\Omega_1 = \frac{\mu_1 a_1}{\sum_{h=1,2} \mu_h a_h} = \frac{q^k k_1 + b_1}{\sum_{h=1,2} \mu_h (q^k k_h + b_h)} = \frac{\mu_1 (q^k k_1 + b_1)}{q^k} \quad (3.10)$$

The formal definition of the Markov equilibrium is given as follows:

Definition 4 . Stationary Markov Equilibrium:

The stationary Markov equilibrium is a collection of each agent h 's consumption decisions $c^h(\Omega_1, s)$, bond holdings $b^h(\Omega_1, s)$, capital holdings $k^h(\Omega_1, s)$, labor hiring $l^h(\Omega_1, s)$, the bond price $q^b(\Omega_1, s)$, the price of capital $q^k(\Omega_1, s)$, and the wealth distribution characterized by Ω_1 , such that:

- (1). *For each agent h , the decision rule $(c^h(\Omega_1, s), b^h(\Omega_1, s), k^h(\Omega_1, s))$ solves the agent's utility maximization problem defined by equations (6) through (9).*
- (2). *The bond market clears: $\sum_h b^h(\Omega_1, s) = 0$*
- (3). *The capital market clears: $\sum_h k^h(\Omega_1, s) = 1$*
- (4). *The labor market clears: $\sum_h l^h(\Omega_1, s) = \sum_h e^h(s)$*

3.4 Parameters and Characterization of Equilibrium

3.4.1 Calibration

The quarterly time series data covers the entire era of great moderation from mid-1980s to 2012. To distinguish the statistics corresponding to episodes of “financial disruption” with those corresponding to “normal cycles”, the data series are split into two sets associated with these two regimes. The event windows for “financial disruption” are based on the NBER-dated contractions associated with severe financial downturns, while other time periods are considered “normal cycles”.⁷

Because the mechanism of the model is highly endogenous, the number of parameters needed to be calibrated is relatively small. The risk-aversion parameter γ is set to standard values widely adopted in the literature. Parameter ϕ can be mapped to the target using the equilibrium condition. Other parameters are calibrated jointly using the simulated method of moments. The model is simulated at bi-quarterly frequency. Table 3.1 summarizes all the parameter values and how they are determined.

Both types of agents have identical preferences. For the utility parameter, the coefficient of risk-aversion in CRRA utility γ is set as the standard value 2, which equals the value used in Kubler and Schmmedders (2003).

The total population is normalized to 1. The 2012 Global Entrepreneur-

⁷During the great moderation era, the U.S. economic contractions associated with severe financial downturns include the 1989-91 U.S. saving-and-loan crisis and the 2008-2010 financial crisis.

Table 3.1: Parameter Values and Targeted Statistics

Variable	Value	Targets or Sources
ϕ	0.3	Capital share during normal cycles is 30%
γ	2	Risk aversion parameter value used by Kubler and Schmedders (2003)
$\beta_1,$	0.980	Ergodic probability of crises regime
$\beta_2,$	0.983	Average short run T-bill rate
μ_1	0.2	Fraction of workforce engaged in entrepreneurship by the 2012 Global Entrepreneurship Monitor U.S. Report
ϵ	0.6	Average leverage ratio is around 0.5 as in BGG
λ	0.8	Averaged -5% below trend during financial disruption
$\theta(s_1)/\theta(s_2)$	0.971	Joint Match the AR1 process of GDP growth
p	0.613	Joint Match the AR1 process of GDP growth

ship Monitor U.S. Report published by Babson College and the London Business School estimates that, in the United States, approximately 15–20 percent of adults in the workforce in each age group are engaged in entrepreneurship. Therefore, the population measure of the expert μ_1 is set to 0.2.

The return-to-scale parameter for capital ϕ is set to match the observed capital share (30%) in the normal steady state. Each agent is assumed to receive the same per capita individual labor endowment and is normalized to 1. Hence, each type of agent supplies labor that equals the population measure for its type, that is, $e^h(s^t) = \mu^h$.

The share of capital that can be used as collateral ϵ is set to 0.6 so that the implied mean leverage ratio is 0.5, the approximate value in the data and also the target value in BGG.

Recall that the two types of agents differ in their ability to manage production. The ordinaries achieve lower productivity than the experts. The parameter of the efficiency gap λ is set to 0.8 to match the average percentage decline in GDP during economic crises.

Because of the endogenous risk, the same exogenous shocks can cause aggregate fluctuations of varying persistency and amplitude. To highlight this point, the shock process is deliberately kept simple. The only exogenous shock is standard productivity shock. There are only two levels of productivity output, the difference between which is within the normal business cycle fluctuation of the U.S. economy. That is, all rare disastrous contractions are endogenous.

The AR1 process of productivity shocks is discretized using the Gauss-Hermite polynomial, the implied two-state transitioning Markov matrix is:

$$\begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix} = \begin{bmatrix} 0.613 & 0.387 \\ 0.387 & 0.613 \end{bmatrix}$$

All calibrations are conducted jointly using the Simulated Method of Moments. Although the above example is highly stylized, it matches some key features of U.S. data. The resulting statistics seem reasonable (see Table 3.2).

Table 3.2: Simulated Statistics and Target Values

Variable Names	Data	Model
Whole Time		
Std of GDP growth	0.015	0.014
Autocorrelation of GDP growth	0.26	0.29
Average GDP growth rate	1.34%	1.34%
Normal Cycles		
Average GDP growth rate	1.55%	1.54%
Ergodic probability of th regime	82%	82%
Average Duration (quarter)	27	27
Financial Disruptions		
Average GDP growth rate	-1.20%	-1.18%
Ergodic probability of th regime	18%	18%
Average Duration (quarter)	6	6

3.4.2 Characterization of Markov Stationary Equilibrium

Policy Function and Portfolio Choices:

I begin with a discussion of the policy functions of each type of agent. As shown in the lower-left panel of Figure 3.1, when high-productivity agents have low wealth as a share of total wealth, they store all their wealth in the form of capital and borrow as much as they can from low-productivity agents. High-productivity agents begin reducing their leverage by gradually reducing their debt obligations only after they obtain all the available capital (see the lower-right panel of Figure 3.1). This is a natural market outcome (consistent with the result of Lemma 2) as the return on capital is higher when it is owned by

high-productivity agents. As shown in the left panel of Figure 3.2, during normal times (i.e., when the economy is not financially constrained), capital returns are excessive compared with bond returns for high-productivity agents, whereas the opposite is true for low-productivity agents. In other words, high-productivity agents prefer capital over bonds whereas low-productivity agents prefer bonds over capital; thus, high-productivity agents are natural holders of capital while low-productivity agents are natural holders of bonds.

Equilibrium Output, Wage, Capital Price and Bond Price:

Based on this understanding of the portfolio choices of the two types of agents, the equilibrium prices and output as functions of endogenous wealth distribution and exogenous productivity shocks can be explained.

The numerical example shows that GDP and the equilibrium wage both increase with the share of capital owned by high-productivity agents. This is an intuitive result insofar as the return on capital is higher when high-productivity agents invest it. Combining this result with agents' capital holding decisions establishes the relationship between economic output and high-productivity agents' wealth. As demonstrated in the lower panel of Figure 3.3, the wealth of high-productivity agents affects aggregate output only when they are financially constrained.

When high-productivity agents are unconstrained, they already own all the productive capital and thus further increasing the wealth of high-productivity agents will not improve production efficiency. On the other hand, when high-productivity agents are financially constrained, they no longer have enough wealth to keep all their capital stocks (i.e., $\mu_1 k < 1$). Some of the capital is

therefore liquidated to and invested by low-productivity agents, resulting in inefficient allocation of resources. Consequently, economic output is lower than its level of efficiency $\theta(s)$ (see the lower-left panel of Figure 3.3).

Equilibrium capital prices, like output and wages, also increase as high-productivity agents take a greater share of total wealth. The mechanism for this is somewhat intuitive: since capital is valued more by high-productivity agents when their share of the wealth is higher, more of the economy's wealth is devoted to acquiring capital stock, driving capital prices up. When the wealth of high-productivity agents is financially constrained, the price of capital must decline to induce low-productivity agents to acquire capital from high-productivity agents (see the upper-left panel of Figure 3.3).⁸ Unlike output and wages, an increase in high-productivity agents' wealth may further raise the price of capital even if they already controlled all the capital because, when high-productivity agents reduce their leverage, the economy is more resilient to adverse shocks and therefore the risk of holding capital endogenously decreases, making capital a more favorable asset.⁹

The feedback loop between capital prices and wealth

As in the upper-left panel of Figure 3.3, the price of capital increases (decreases) when the share in the wealth of high-productivity agents increases (decreases). Meanwhile, any change in capital prices feeds back to the distribution of wealth. Recall that the evolution of the wealth distribution is characterized by the transitioning function $\Omega'_1(\Omega_1, s, s')$ that maps the current wealth share of

⁸The price of capital also depends on productivity realization because it affects the return on capital.

⁹At the individual level, every high-productivity agent also has a more comfortable wealth cushion to protect against risks.

high-productivity agents Ω_1 to their wealth share in the next period Ω'_1 . Specifically:

$$\Omega'_1(\Omega_1, s, s') = \frac{\mu_1[q^k(\Omega'_1, s')k'_1(\Omega_1, s) + b_1(\Omega_1, s)]}{q^k(\Omega'_1, s')} = \mu_1[k'_1(\Omega_1, s) + \frac{b_1(\Omega_1, s)}{q^k(\Omega'_1, s')}] \quad (3.11)$$

As shown on the right-hand-side of equation (11), the future wealth share depends on the future capital position $k'_1(\Omega_1, s)$, future bond payments $b_1(\Omega_1, s)$, and the future price of capital $q^k(\Omega'_1, s')$. Since high-productivity agents always have a negative bond position in the stochastic steady state distribution (i.e. $b_1(\Omega_1, s) < 0$), a rise in the future price of capital will increase the share of the wealth for high-productivity agents while a drop in the future price of capital will decrease that share. Future productivity output s' affects only future capital prices, whereas future capital positions $k'_1(\Omega_1, s)$ and future bond payments $b_1(\Omega_1, s)$ are already determined for the current period; therefore, $q^k(\Omega'_1, s_{low}) < q^k(\Omega'_1, s_{high}) \Rightarrow \Omega'_1(\Omega_1, s, s_{low}) < \Omega'_1(\Omega_1, s, s_{high})$.

Figure 3.4 illustrates the evolution of the wealth share. Given a bad shock in the next period, Ω'_1 falls below the 45-degree line, implying a decrease in the wealth share. In comparison, given a good shock in the next period, Ω'_1 rises above the 45-degree line, implying an increase in the share of wealth.

This reinforcing loop between wealth distribution and capital price could amplify macro shocks, especially when high-productivity agents are financially constrained in a crisis regime. In that case, a downside shock s_{low} directly represses output and capital prices through lower productivity $\theta(s_{low})$ and indirectly through a lower share of high productivity agents' wealth $\Omega'_1(\Omega_1, s, s_{low})$. In contrast, when the leverage of high-productivity agents is far below its endogenous upper limit, a drop in $\Omega'_1(\Omega_1, s, s_{low})$ will not lead to a further decline in output as high-productivity agents will retain all their capital stocks. In this

case, the vicious macro-financial linkage breaks.

Endogenous Upper-Limit of Leverage

In equilibrium, the financial market anticipates the endogenous increase in future risk and responds to it by imposing a more restrictive requirement on leverage. Following Geanakoplos (2010), the leverage ratio is defined as the current market value of capital $q^k k_1$ divided by the portion of this value that is self-financed $q^k k_1 + b_1$. The borrowing constraint in equation (9) implies that the upper limit of leverage $Lev_{max}(\Omega_1, s)$ that a borrower can reach is:

$$Lev_{max}(\Omega_1, s) = \frac{q^k(\Omega_1, s)}{q^k(\Omega_1, s) - \epsilon \min_{s'}(q^k(\Omega'_1, s'))} \quad (3.12)$$

In comparison, the equilibrium leverage of high-productivity agents is:

$$Lev_1(\Omega_1, s) = \frac{q^k(\Omega_1, s)k'_1(\Omega_1, s)}{q^k(\Omega_1, s)k'_1(\Omega_1, s) + b'_1(\Omega_1, s)} \quad (3.13)$$

As shown in Figure 3.5, the endogenous upper limit of leverage decreases when the equilibrium leverage becomes “too” high as the financial market anticipates a potential plummet in the future price of capital. It is worth noting that such endogenous tightening of the collateral requirement cannot prevent financial disruptions. Nonetheless, this rational and forward-looking behavior on the part of creditors protects them from massive default when financial disruptions occur.

Once high-productivity agents become financially constrained, they are no longer able to keep all their capital stocks, depressing the price of capital and output. To measure the impact of such a credit crunch on capital investment, the excess return on capital $E(r_t^k) - E(r_t)$ and the Sharpe Ratio SR_t are computed

according to:

$$E(r_h^k - r) = \frac{q'^k}{q - \theta_h \phi(k'_h/l_h)^{\phi-1}}$$

and

$$SR_h = \frac{E(r_h^k - r)}{\sigma(r_h^k - r)}$$

As shown in Figure 3.2, the risk premium on as well as the Sharpe Ratio associated with capital investment sharply increases for financially constrained agents.

Having discussed the basic mechanism of the model, I now highlight some of its key features.

3.5 Model Features and the Simulated Results

Asymmetric Response to Adverse Shocks (Normal Cycles vs. Financial Disruptions):

Because the system's reaction to shocks is highly nonlinear, the model is able to clearly distinguish between normal business cycles and episodes of financial disruption. To highlight the differences between normal recessions and recessions associated with financial disruptions, I conduct two controlled simulations using the same sequence of shocks (four periods of bad shocks followed by a good shock). The only difference between the two simulations is in the initial endogenous state. In the first simulation (the blue line), the initial $\Omega_1 = 0.43$, which implies that the leverage ratio of high-productivity agents is very close to the endogenous maximum, which implies that they will be financially constrained following a bad shock, triggering the financial disruption. In

the second simulation (the red line), initial $\Omega_1 = 0.55$, which implies that the leverage ratio of high-productivity agents is sufficiently lower than the endogenous maximum to enable them to ride out the sequence of bad shocks without hitting the debt limit. Therefore, the first simulation corresponds to episodes of financial disruption whereas the second simulation corresponds to recessions during normal cycles.

During normal cycles, the system disturbance remains close to the center of the stochastic steady state distribution whereby the economy enjoys efficient capital allocation, that is, all capital is controlled by high-productivity agents. In contrast, during a financial disruption, high-productivity agents share in the wealth is depressed, so they will have to sell capital to low-productivity agents, pushing the economy into misallocation of resources (see the lower-mid panel of Figure 3.6)

Recall that the output of the economy (GDP) depends on two elements: the exogenous productivity shock and the endogenous allocation of capital. During normal cycles, recessions are driven by adverse productivity shocks. In comparison, during financial disruptions the economy suffers not only from exogenous downside productivity shocks but also from endogenous loss of efficiency due to misallocation of capital. Thus, recessions associated with financial disruptions are stronger and longer-lasting than typical recessions during normal cycles (see the upper-left and upper-mid panels of Figure 3.6).

As shown in the upper-right and lower-left panels of Figure 3.6, the risk premium on capital as well as the Sharpe Ratio rise significantly during financial disruptions as high-productivity agents are financially constrained.¹⁰ This result

¹⁰Note that the risk premium and Sharpe Ratio also increase, but only slightly, during normal

resonates with what was observed in the 2008 financial crisis and is consistent with the results reported in He and Krishnamurthy (2012).

We can see the asymmetry of the aggregate leverage in the lower-right panel of Figure 3.6. During normal cycles, high-productivity agents absorb adverse shocks through collateral borrowing. However, during financial disruptions high-productivity agents have to borrow less instead of borrowing more, due to the collapse in value of the collateral. In addition, in these circumstances high-productivity agents lose some capital, further depressing their ability to commit to new loans. Consequently, aggregate lending is depressed.

Endogenous Risk of “Financial Disruption”:

Because the collateral constraint binds only occasionally, the likelihood of which changes the entire state-space, “financial disruption” is an endogenous, low-probability event. The endogenous risk of “financial disruption” manifests itself in the varying expected future volatility of the economy. Figure 3.7 plots the conditional variance of next-period capital prices (left panel) and GDP (right panel). Here we can clearly see that, when high-productivity agents share of the wealth (and so their absolute wealth) is high (i.e., the leverage ratio is far below the endogenous upper limit), aggregate variables such as GDP and capital prices are fairly stable. In this case, the risk of “financial disruption” is low as high-productivity agents are able to absorb bad shocks using their comfortable wealth cushion and stay at the optimal investment level. In contrast, when high productivity agents wealth is low and their leverage ratio is approaching the endogenous upper limit, even a small adverse shock could trigger an episode of “financial disruption.” In that case, capital prices and GDP are knocked off

recessions. This is in large degree due to the persistency of the exogenous shock process.

balance as high-productivity agents are less able to cushion the shocks.

This feature of the model is consistent with the stylized empirical facts that economic volatility is counter-cyclical and volatility has a systematic negative relationship with agents' wealth.¹¹ In addition, a simulation covering 1000 periods reveals wide, low-frequency output volatility swings associated with gradual changes in aggregate wealth.

Uncertain Trough:

Financial frictions prolong the impact of a shock on the economy as it takes time for levered agents to rebuild their wealth after the shock. Once an economy is hit by an exogenous shock, it may not be able to rebound to its normal state immediately. The depth of the recession and the length of the recovery depend on the liquidity of high-productivity agents. If the economy were already in a volatile regime with high leverage, a subsequent negative shock can push the economy further into crisis. In this sense, the trough is uncertain and so is the duration of the contraction.

Figure 3.9 plots an episode of financial disruption of median depth relative to all financial disruptions simulated over 10,000 periods. It demonstrates that, once the economy enters an episode of financial disruption (corresponding to the area below the red dashed line), any subsequent bad shock will push the economy further into contraction. Moreover, the economy may not be able to rebound to the unconstrained state after a good shock. Once it enters an episode of financial disruption, the system is vulnerable to shocks and the economy is

¹¹For example, Heathcote and Perri (2013) find that in the U.S. over the course of the past century there has been a systematic negative relationship between household wealth and business cycle volatility.

prone to a multiple-dip recession.

As a result, the stationary distribution takes a double-humped shaped, with one peak near the center of the steady state distribution and the other at the left-side tail of the distribution (see Figure 3.10). This suggests that high-productivity agents remain financially depressed and the economy may be stuck in crisis mode for quite a long time once it sets in.

3.6 Conclusion

In this paper, I study the full equilibrium dynamics of an economy with two distinct regimes, “normal cycles” and rare episodes of “financial disruption.” I develop a model in which two types of households with differing levels of productivity coexist in an infinite-horizon production economy. Because the return on capital is higher when it is invested by high-productivity agents, they are the natural holders of capital. Due to the debt enforcement problem, however, there is a borrowing limit, which depends on the market’s projection of the future value of capital put up as collateral. Because of the precautionary motive, the borrowing limit binds occasionally, triggering financial disruption, while the economy spends most of its time in unconstrained normal cycles.

Since the two types of agents differing regarding the return on their investment, the distribution of wealth affects the true underlying state of the economy. Interaction between the two types of agents in the entire state-space equips the model with several features such as endogenous risk, asymmetric response to shock, endogenous credit crunches, and uncertain troughs. These features allow my model to achieve the following:

1. The model characterizes two distinct regimes over business cycles and highlights their differences. During normal cycles, high-productivity agents are able to maintain an optimal level of capital stocks and absorb adverse shocks through collateral borrowing. In episodes of financial disruption, such high-productivity agents are financially constrained, in which case they have to sell capital to low-productivity agents, leading to lower capital prices. The decline in the price of capital generates an adverse feedback loop by shrinking the value of collateral, further suppressing borrowing capacity. Due to this highly nonlinear spiral effect, financial disruptions have unique features such as Fisher debt deflation, sudden declines in both output and capital prices, and increases in both the risk premium and Sharpe Ratios associated with capital. Moreover, given the time it takes for levered agents to rebuild their wealth, a recovery might be slow and the scale of the slump is uncertain.

2. The model illustrates how business cycles and financial disruptions interact. On the one hand, financial disruptions have a strong negative impact on business cycles as they result in the misallocation of capital to low-productivity agents, reducing the efficiency of production. This explains a well established empirical regularity according to which recessions associated with financial disruptions tend to be stronger and deeper than typical recessions during normal cycles. On the other hand, financial disruptions are equilibrium outcomes evolving with business cycles: the probability of regime switching depends on the history of macro shocks that affect equilibrium leverage. In other words, the model shows that major turmoil in real economic and financial markets is not necessarily caused by rare disastrous events. As equilibrium leverage determines the scale of contraction, even small shocks could sporadically trigger large contractions. Moreover, the financial market responds endogenously to

changes in business conditions. In other words, the credit crunches and deleveraging that are typical of crises could be an outcome of rational expectations as the financial market anticipates future adjustments in asset prices when there is a business slowdown.

The model emphasizes the importance of wealth in linking the financial market with the real economy. The only market friction is the possibility of default, and therefore collateral is required to obtain loans. When the equilibrium leverage of the financial market is high, the market tends to be instable: once high-productivity agents lose portions of their wealth, the financial market might be unable to allocate funds to the most productive users, leading to a joint decline in the price of capital and output. To highlight this credit market friction while controlling for other possible principle-agency problems, the mechanism of the model operates in a relatively simple environment in which households are the ultimate owners of the economy. In this way I embed firms' financing decisions in those of firm owners.

Insofar as economic crises are endogenous equilibrium scenarios, I believe this model is useful for discussing the potential role of government policies in preventing future financial crises and simulating recovery. I leave these discussions to future studies.

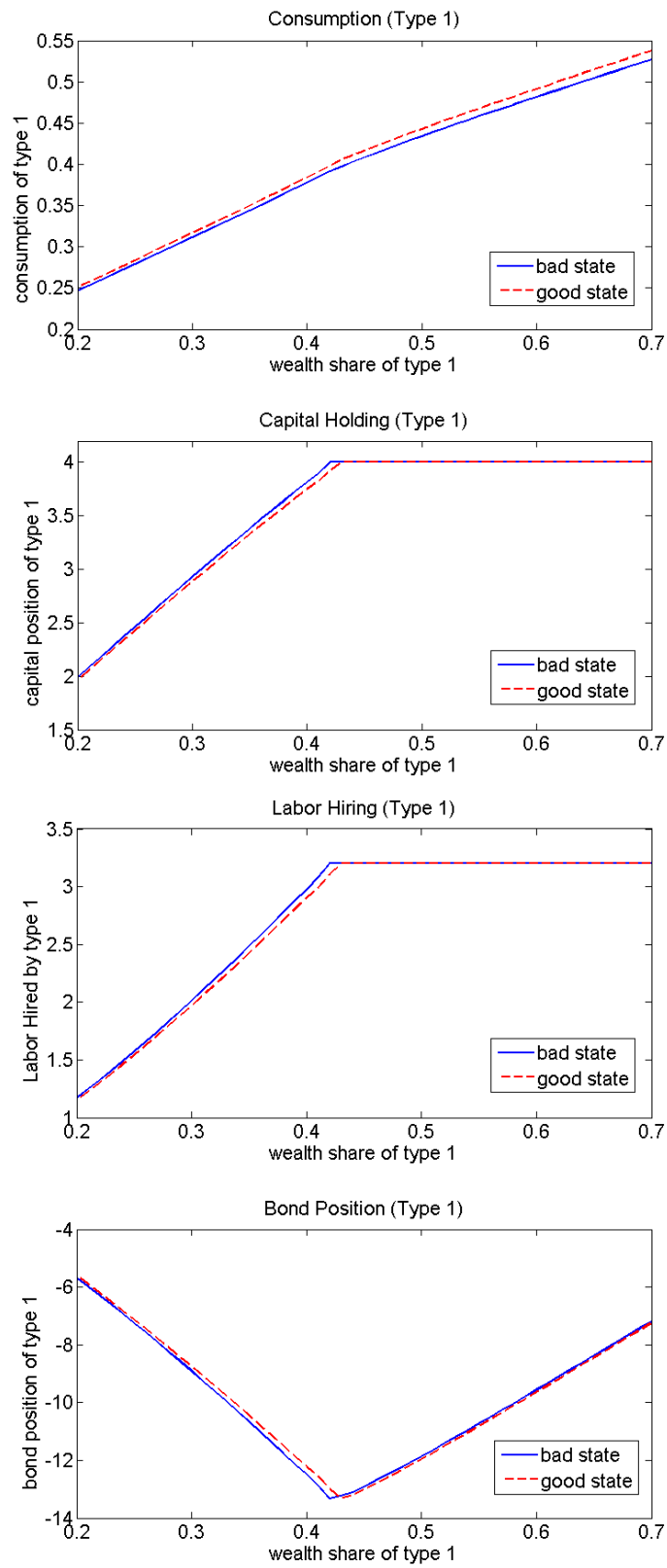


Figure 3.1: Policy Plot

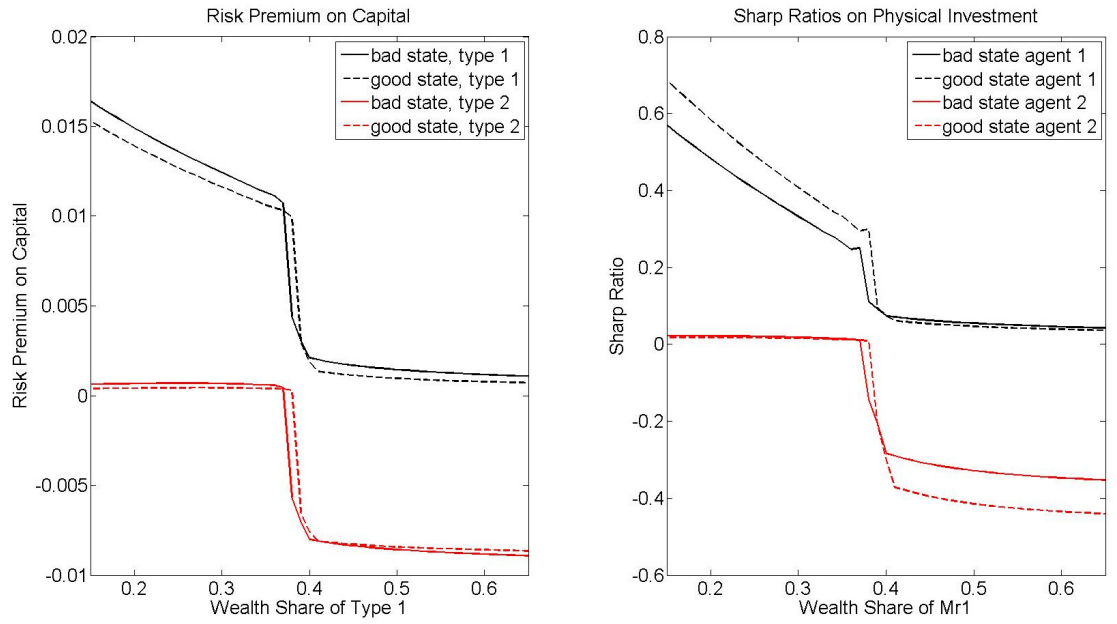


Figure 3.2: Risk Premium and Sharpe Ratios

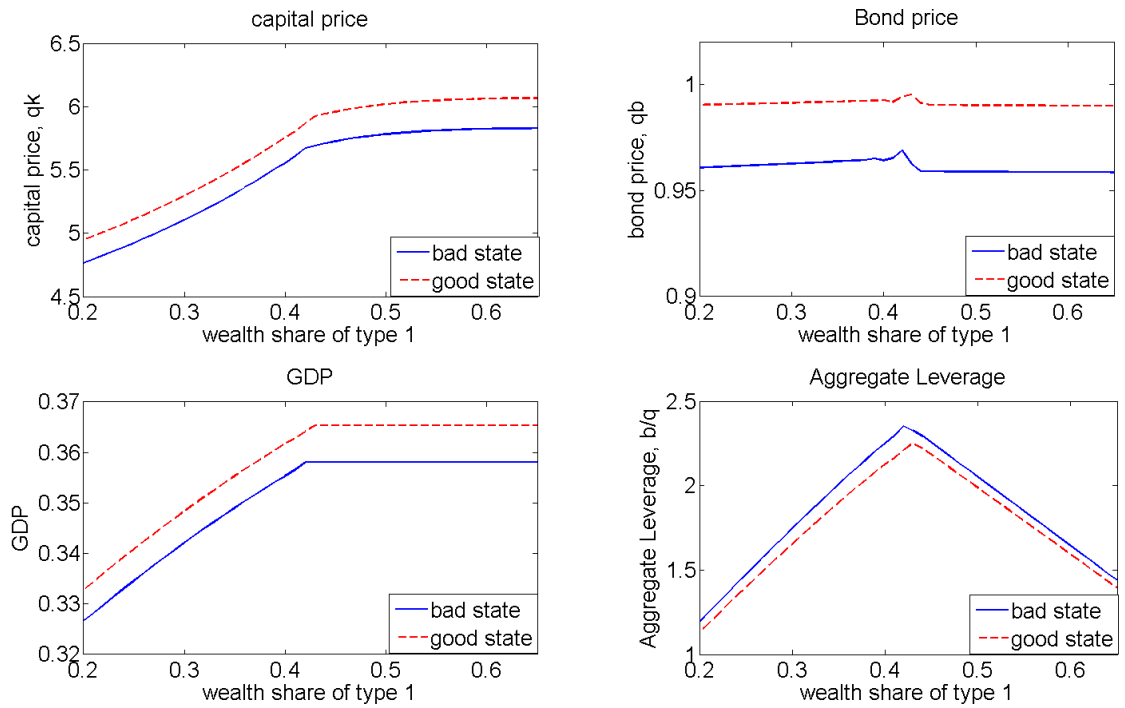


Figure 3.3: Aggregate Prices, Output and Leverage

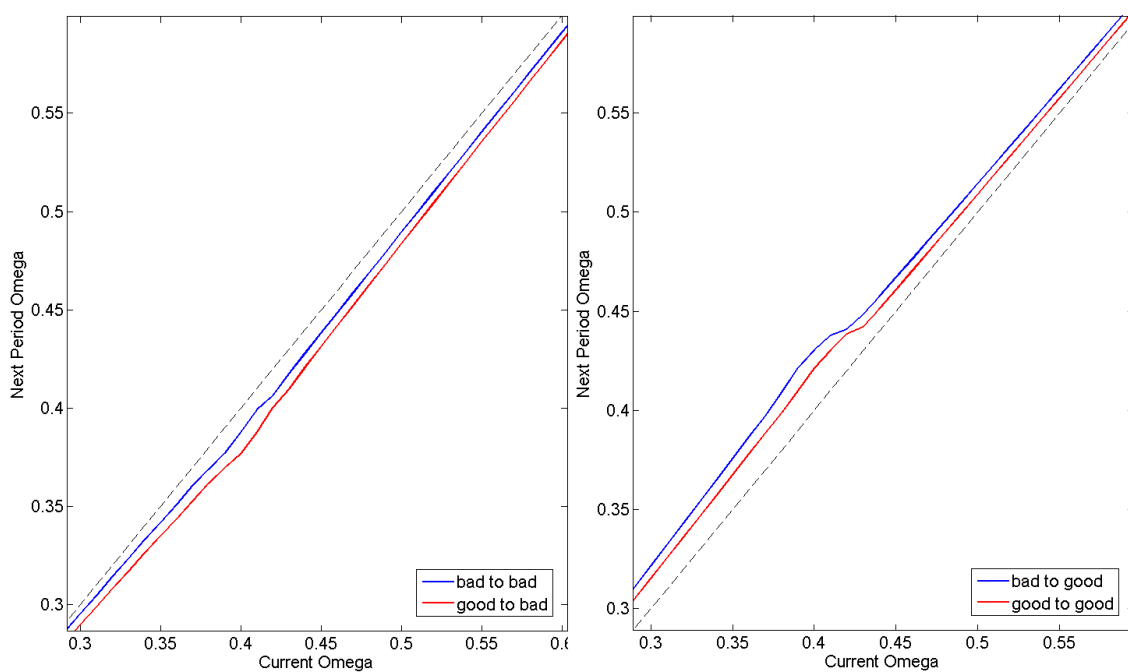


Figure 3.4: Evolution of the Endogenous State Variable Ω_1

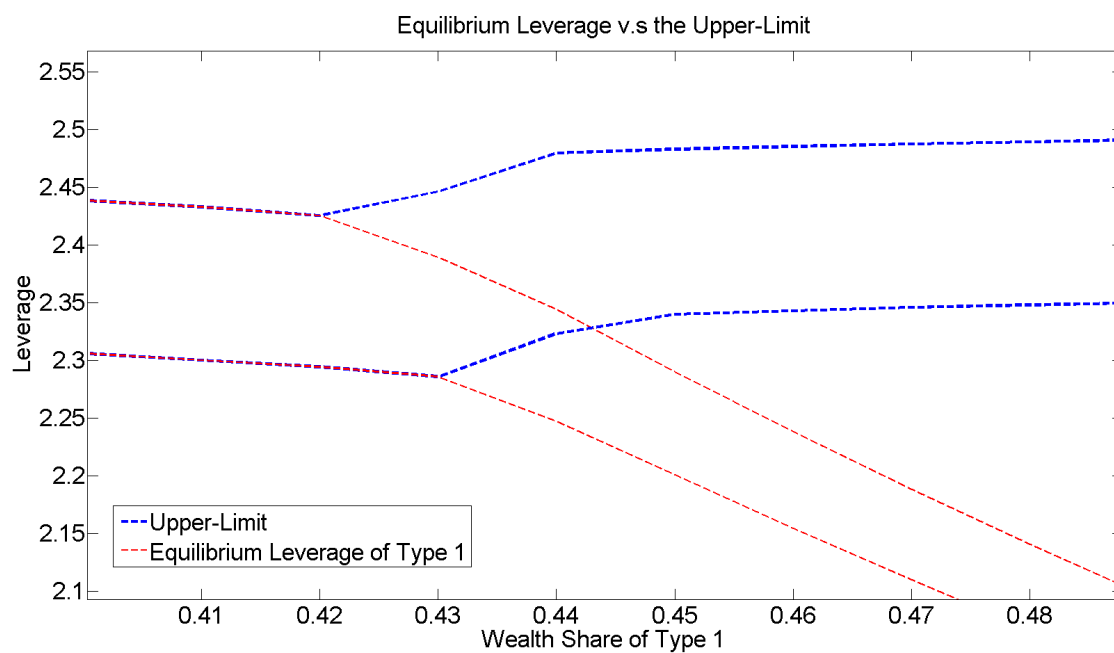


Figure 3.5: Equilibrium Leverage vs. the Upper Limit

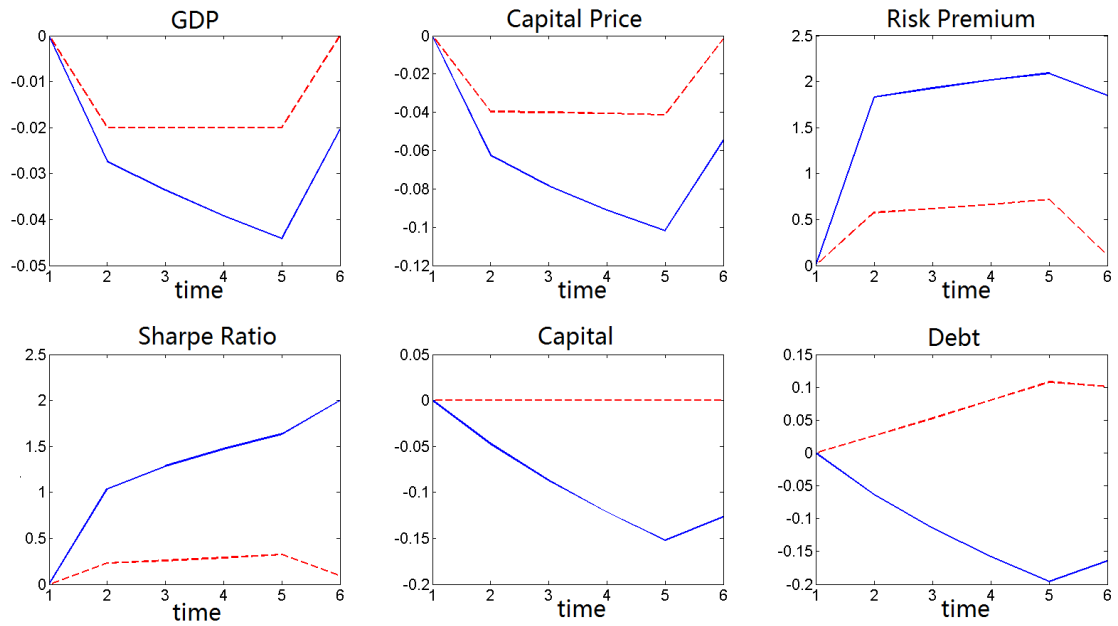


Figure 3.6: Asymmetric Response in Cycles

Dashed line plots normal cycles; solid line plots financial disruption.

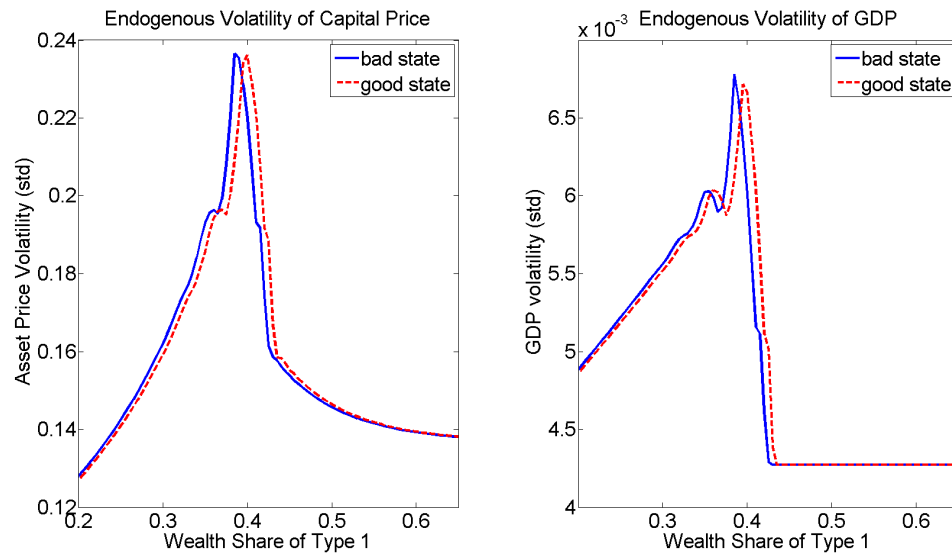


Figure 3.7: Endogenous Volatility of the Economy

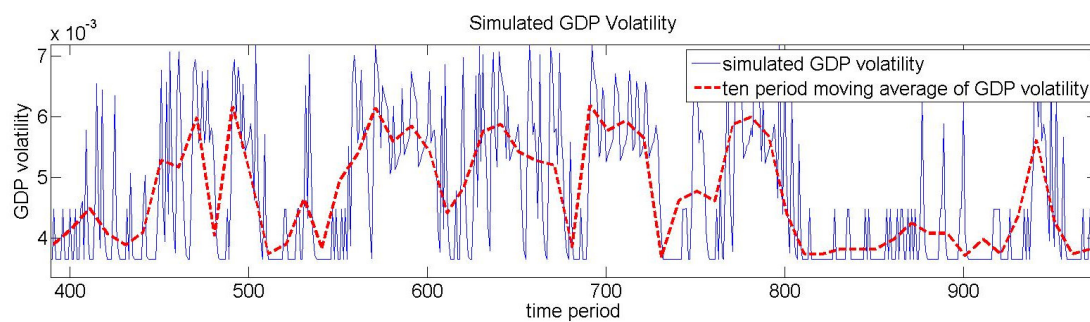


Figure 3.8: Simulated GDP Volatility of an Economy

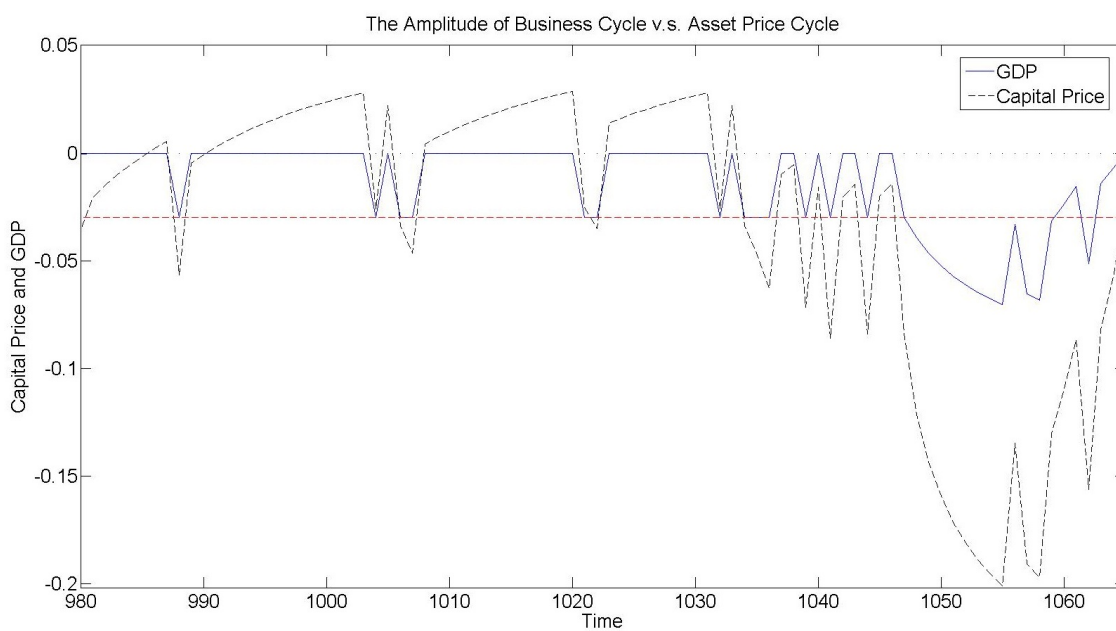


Figure 3.9: Uncertain Trough

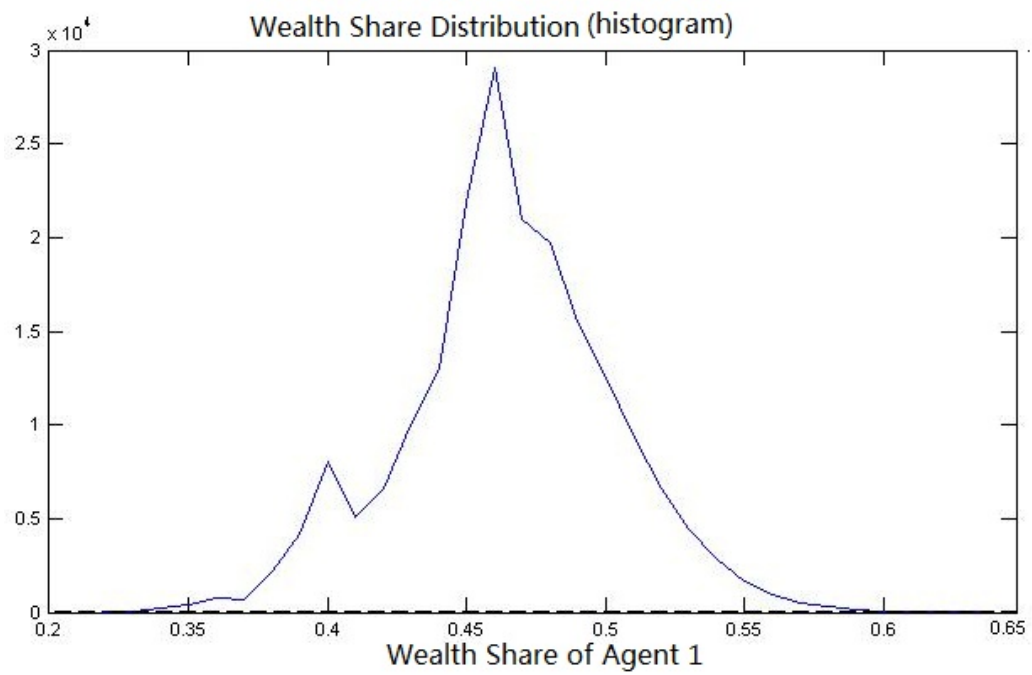


Figure 3.10: Histogram: Stationary Distribution of High-Productivity Agents' Wealth Share

APPENDIX A
APPENDIX FOR ESSAY ONE

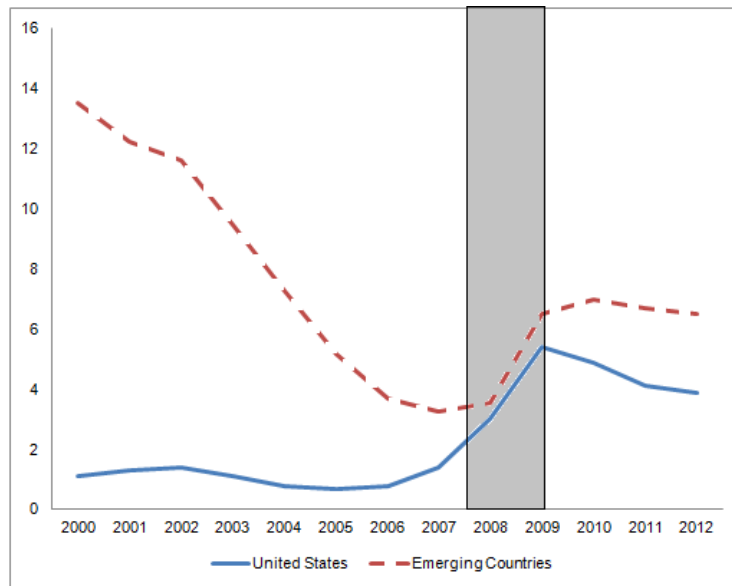


Figure A.1: Non-Performing Loan Ratio for the U.S and Emerging Countries.

Source: WDI and FRED database.

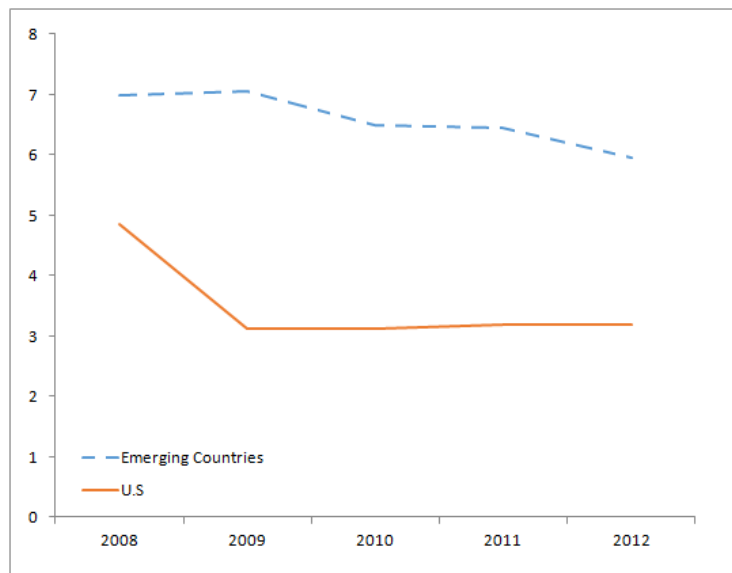


Figure A.2: Loan-deposit spread for the U.S. and Emerging Countries.

Source: WDI and FRED database.

Figure A.1 shows the Non-Performing Loan (NPL) ratio for the U.S. and for emerging countries. (“Emerging Countries” are selected according to the IMF definition.) Note that NPL ratios are persistently higher in emerging countries than in the United States. Although this difference narrowed during the U.S financial crisis (gray area), it widened again after the crisis.

Figure A.2 plots the loan-deposit spread for the U.S. and for emerging countries. (“Emerging Countries” are selected according to the IMF definition.) Note that the loan-deposit spread is persistently in emerging countries than in the United States.

APPENDIX B

APPENDIX FOR ESSAY THREE: REMOVING THE ASSUMPTION ON HETEROGENEOUS DISCOUNTING FACTORS

The figure below plots the equilibrium aggregate variables with and without the assumption of heterogeneous discounting factor. As demonstrated in Figure B.1, the model mechanisms hold without the assumption of heterogeneous discounting factor.

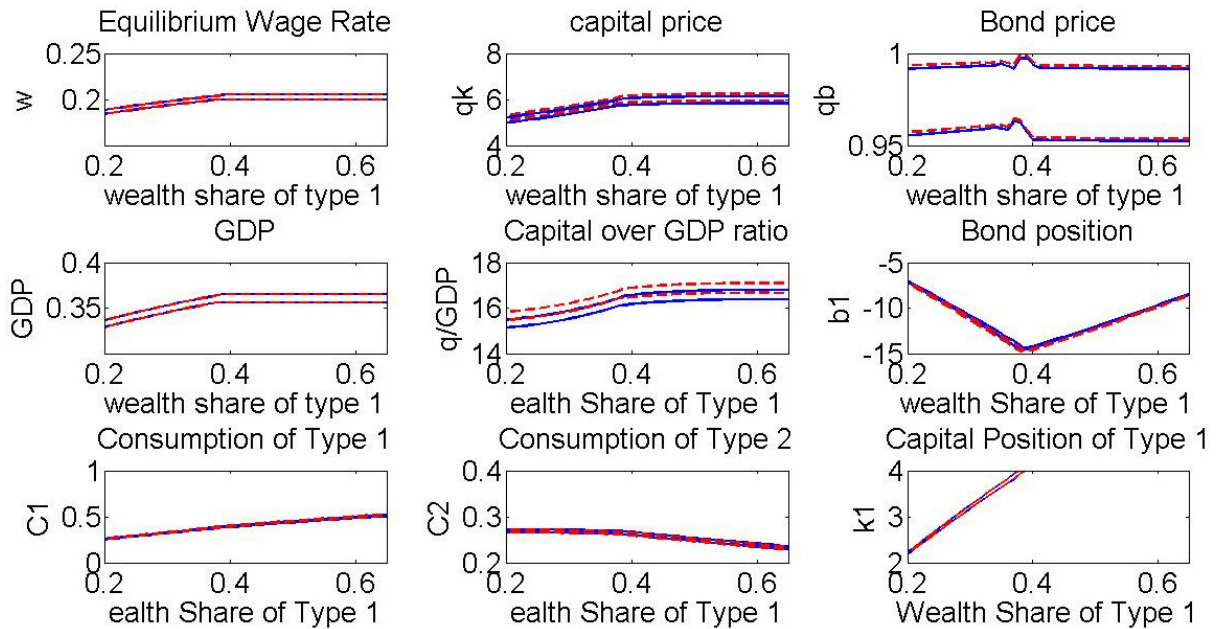


Figure B.1: Comparison between Models with and without the Assumption of Heterogeneous Discounting Factor.

With heterogeneous discounting factor (red line); Without heterogeneous discounting factor (blue line).

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